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Transition Energy Field and Correlation Equations

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Introduction. The method of correlation equations is one of the most demanded means of constructing and studying systems of mathematical statistical physics in infinite volumes (see, for example, [2, 6, 7, 9, 10]). In the case of lattice systems, this method has generally been applied to vacuum systems with two states: spin and vacuum.

The problem of extending the applied method of correlation equations for more general systems naturally arises. This problem was considered in [8], in which a measurable set of finite measure was considered as a spin space, and the vacuum measure was taken to be equal to unity. In all mentioned papers, the definition of correlation function is based on the notion of interaction potential.

In the present work, we consider systems with finite spin space. Based on the results of [1, 3–5], a system of correlation equations is written using the concept of the transition energy field introduced in [1]. It is shown that for a sufficiently small value of the one-point transition energies, the corresponding system of correlation functions, considered in infinite space, has a solution which is unique.

1. Preliminaries. Let $\mathbb{Z}^d$ be a d-dimensional integer lattice, i.e., a set of d-dimensional vectors with integer components, $d \geq 1$. Note that all the arguments in this paper remain valid if we consider an arbitrary countable set instead of $\mathbb{Z}^d$.

For $S \subset \mathbb{Z}^d$, denote by $W(S) = \{\Lambda \subset S, |\Lambda| < \infty\}$ the set of all finite subsets of S, where $|\Lambda|$ is the number of points in Λ. In the case $S = \mathbb{Z}^d$, we will use the simpler notation W. To denote the complement of the set S, we will write S^c. For one-point sets $\{t\}$, $t \in \mathbb{Z}^d$, braces will be omitted. For $t = (t^{(1)}, t^{(2)}, \dots, t^{(d)}), s = (s^{(1)}, s^{(2)}, \dots, s^{(d)}) \in \mathbb{Z}^d$, we denote $|t - s| = \max_{1 \le i \le d} |t^{(i)} - s^{(i)}|$.

Let each point $t \in \mathbb{Z}^d$ be associated with a set X^t, which is a copy of some finite set X, $1 < |X| < \infty$. Denote by X^S the set of all configurations on S, $S \subset \mathbb{Z}^d$, that is, the set $X^S = \{x = (x_t, t \in S), x_t \in X\}$ of all functions defined on S and tacking values in X. For $S = \emptyset$, we assume that $X^{\emptyset} = \{\emptyset\}$ where $\emptyset$ is an empty configuration. For any disjoint $S, T \subset \mathbb{Z}^d$ and any $x \in X^S$, $y \in X^T$, denote by xy the concatenation of x and y, that is, the configuration on $S \cup T$ equal to x on S and to y on T. When $T \subset S$, we denote by x_T the restriction of configuration $x \in X^S$ on T, i.e., $x_T = (x_t, t \in T)$.

Let θ_t be some fixed element of X^t (vacuum) and $\theta = \{\theta_t, t \in \mathbb{Z}^d\}$. Denote $X_*^t = X^t \setminus \theta_t$, $t \in \mathbb{Z}^d$. For any $S \subset \mathbb{Z}^d$, denote by X_*^S the set of configurations on S which components do not contain the vacuum, and let $L_*^S = \bigcup_{J \in W(S)} X_*^J$ be the set of configurations without vacuum which supports are subsets of S. In the case $S = \mathbb{Z}^d$, we denote $L_* = L_*^{\mathbb{Z}^d}$. Note that any configuration from X^S can be written as $x\theta_{S \setminus I}$ where $x \in X_*^I$, $I \subset S$. It is not difficult to see that $X^S = \bigcup_{I \subset S} \{x\theta_{S \setminus I}, x \in X_*^I\}$.

Finally, for any $S \subset \mathbb{Z}^d$ and any function $h: W(S) \to \mathbb{R}$, the notation $\lim_{\Lambda \uparrow \mathbb{Z}^d} h(\Lambda) = a$ means that for any $\varepsilon > 0$, there exists $\Lambda_\varepsilon \in W(S)$ such that for any $\Lambda \in W(S)$, $\Lambda \supset \Lambda_\varepsilon$, it holds $|h(\Lambda) - a| < \varepsilon$.

2. Transition energy fields. In [1], the notions of transition energy field and one-point transition energy field were introduced.

A set $\Delta = \{\Delta_\Lambda^{\bar{x}}, \bar{x} \in X^{\Lambda^c}, \Lambda \in W\}$ of functions $\Delta_\Lambda^{\bar{x}}(x, u)$, $x, u \in X^\Lambda$, is called *transition energy field* if its elements satisfy the following consistency conditions: for all $\Lambda \in W$ and $\bar{x} \in X^{\Lambda^c}$, it holds

$$\Delta_\Lambda^{\bar{x}}(x, u) = \Delta_\Lambda^{\bar{x}}(x, z) + \Delta_\Lambda^{\bar{x}}(z, u), \qquad x, u, z \in X^\Lambda;$$

and for all disjoint $\Lambda, V \in W$ and $\bar{x} \in X^{(\Lambda \cup V)^c}$,

$$\Delta_{\Lambda \cup V}^{\bar{x}}(xy, uv) = \Delta_\Lambda^{\bar{x}y}(x, u) + \Delta_V^{\bar{x}u}(y, v), \qquad x, u \in X^\Lambda, y, v \in X^V.$$

Note that in particular, it holds

$$\Delta_\Lambda^{\bar{x}}(x, u) = -\Delta_\Lambda^{\bar{x}}(u, x), \qquad \Delta_\Lambda^{\bar{x}}(x, x) = 0, \qquad x, u \in X^\Lambda.$$

A set $\Delta_1 = \{\Delta_t^{\bar{x}}, \bar{x} \in X^{t^c}, t \in \mathbb{Z}^d\}$ of functions $\Delta_t^{\bar{x}}(x, u)$, $x, u \in X^t$, is called *one-point transition energy field* if its elements satisfy the following consistency conditions: for all $t \in \mathbb{Z}^d$ and $\bar{x} \in X^{t^c}$, it holds

$$\Delta_t^{\bar{x}}(x, u) = \Delta_t^{\bar{x}}(x, z) + \Delta_t^{\bar{x}}(z, u), \qquad x, u, z \in X^t;$$

and for all $t, s \in \mathbb{Z}^d$ and $\bar{x} \in X^{\{t,s\}^c}$,

$$\Delta_t^{\bar{x}y}(x, u) + \Delta_s^{\bar{x}u}(y, v) = \Delta_s^{\bar{x}x}(y, v) + \Delta_t^{\bar{x}v}(x, u), \qquad x, u \in X^t, y, v \in X^s.$$

The following theorem states the relationship between the elements of a transition energy field and a one-point transition energy field (see [1] as well as [4]).

Theorem 1. *A set* $\Delta = \{\Delta_\Lambda^{\bar{x}}, \bar{x} \in X^{\Lambda^c}, \Lambda \in W\}$ *of functions* $X^\Lambda \times X^\Lambda$ *is a transition energy field if and only if its elements can be represented in the form*

$$\Delta_\Lambda^{\bar{x}}(x,u) = \Delta_{t_1}^{\bar{x}x_{\Lambda\setminus t_1}}(x_t, u_{t_1}) + \Delta_{t_2}^{\bar{x}u_{t_1}x_{\Lambda\setminus\{t_1,t_2\}}}(x_{t_2}, u_{t_2}) + \cdots + \Delta_{t_n}^{\bar{x}u_{\Lambda\setminus t_n}}(x_{t_n}, u_{t_n}),$$

$x, u \in X^\Lambda$, *where* $\Lambda = \{t_1, t_2, \dots, t_n\}$ *is some enumeration of points in* Λ, $|\Lambda| = n$, *and* $\Delta_1 = \{\Delta_t^{\bar{x}}, \bar{x} \in X^{t^c}, t \in \mathbb{Z}^d\}$ *is a one-point transition energy field.*

Thus, the one-point transition energy field Δ_1 uniquely determines the transition energy field Δ. Therefore, when obtaining results, conditions can only be imposed on Δ_1.

3. Correlation functions. Let $\Delta_1 = \{\Delta_t^{\bar{x}}, \bar{x} \in X^{t^c}, t \in \mathbb{Z}^d\}$ be a one-point transition energy field and let $\Delta = \{\Delta_\Lambda^{\bar{x}}, \bar{x} \in X^{\Lambda^c}, \Lambda \in W\}$ be the corresponding transition energy field. Let us fix some $\Lambda \in W$. To simplify notations, we denote $\Delta_\Lambda = \Delta_\Lambda^{\theta_{\Lambda^c}}$, and for any $t \in \Lambda$ and $z \in X^{\Lambda\setminus t}$, we will write Δ_t^z instead of $\Delta_t^{z\theta_{\Lambda^c}}$.

Finite-volume correlation function relative to Λ is a function ρ_Λ on L_* defined by

$$\rho_\Lambda(x) = \frac{1}{Z_\Lambda} \sum_{y \in X^{\Lambda\setminus I}} e^{\Delta_\Lambda(xy, \theta_\Lambda)}, \qquad x \in X_*^I, I \subset \Lambda,$$

where

$$Z_\Lambda = \sum_{x \in X^\Lambda} e^{\Delta_\Lambda(x, \theta_\Lambda)},$$

$\rho_\Lambda(\emptyset) = 1$, and $\rho_\Lambda(x) = 0$ if $x \in X_*^I$ and $I \not\subset \Lambda$.

Thus, each Δ_1 defines a set of finite-volume correlation functions $\{\rho_\Lambda, \Lambda \in W\}$. Under a suitable condition on the elements of Δ_1, it can be shown that each finite-volume correlation function satisfies a certain equation.

Theorem 2. *Let* $\Delta_1 = \{\Delta_t^{\bar{x}}, \bar{x} \in X^{t^c}, t \in \mathbb{Z}^d\}$ *be a one-point transition energy field such that for any* $t \in \Lambda \in W$ *and* $x, u \in X^t, y, z \in X^{\Lambda\setminus t}$, $\bar{x} \in X^{\Lambda^c}$, *it holds*

$$\Delta_t^{\bar{x}y}(x, \theta_t) - \Delta_t^{\bar{x}z}(x, \theta_t) = \Delta_t^y(x, \theta_t) - \Delta_t^z(x, \theta_t). \qquad (1)$$

Then for any $\Lambda \in W$, *correlation function* ρ_Λ *satisfies the following equation: for any* $t \in I \subset \Lambda \in W$ *and* $x \in X_*^t$, $u \in X_*^{I\setminus t}$, *it holds*

$$\rho_\Lambda(xu) = \frac{e^{\Delta_t^u(x,\theta_t)}}{\sum_{\alpha \in X^t} e^{\Delta_t^u(\alpha,\theta_t)}} \left(\rho_\Lambda(u) + \sum_{\alpha \in X_*^t} e^{\Delta_t^u(\alpha,\theta_t)} \left(G_\Lambda(xu) - G_\Lambda(\alpha u)\right) \right),$$

where

$$G_{\Lambda}(xu) = \sum_{J\subset\Lambda\setminus I}\sum_{y\in X_*^J} K_{I\cup J}(xy)\left(\rho_{\Lambda}(uy) + \sum_{\alpha\in X_*^t}\rho_{\Lambda}(\alpha uy)\right)$$

and

$$K_{I\cup J}(xy) = \prod_{s\in J}\left(e^{\Delta_s^{xu}(y_s,\theta_s)-\Delta_s^{u}(y_s,\theta_s)} - 1\right).$$

Sketch of the proof. Let $t \in I \subset \Lambda \in W$ and $x \in X_*^t$, $u \in X_*^{I\setminus t}$. Since for any $y \in X^{\Lambda\setminus I}$,

$$\Delta_{\Lambda}(xuy,\theta_{\Lambda}) = \Delta_t^{u}(x,\theta_t) + \Delta_{\Lambda}(\theta_t uy,\theta_{\Lambda}) + \Delta_t^{uy}(x,\theta_t) - \Delta_t^{u}(x,\theta_t),$$

we can write

$$\rho_{\Lambda}(xu) = \frac{1}{Z_{\Lambda}}\sum_{y\in X^{\Lambda\setminus I}} e^{\Delta_{\Lambda}(xuy,\theta_{\Lambda})}$$
$$= \frac{e^{\Delta_t^{u}(x,\theta_t)}}{Z_{\Lambda}}\left(\sum_{y\in X^{\Lambda\setminus I}} e^{\Delta_{\Lambda}(\theta_t uy,\theta_{\Lambda})} + \sum_{y\in X^{\Lambda\setminus I}} e^{\Delta_{\Lambda}(\theta_t uy,\theta_{\Lambda})}\left(e^{\Delta_t^{uy}(x,\theta_t)-\Delta_t^{u}(x,\theta_t)} - 1\right)\right).$$

For the first summand in the obtained relation, we have

$$\frac{1}{Z_{\Lambda}}\sum_{y\in X^{\Lambda\setminus I}} e^{\Delta_{\Lambda}(\theta_t uy,\theta_{\Lambda})} = \rho_{\Lambda}(u) - \sum_{\alpha\in X_*^t}\rho_{\Lambda}(\alpha u).$$

Let us consider the second summand

$$G_{\Lambda}(xu) = \frac{1}{Z_{\Lambda}}\sum_{y\in X^{\Lambda\setminus I}} e^{\Delta_{\Lambda}(\theta_t uy,\theta_{\Lambda})}\left(e^{\Delta_t^{uy}(x,\theta_t)-\Delta_t^{u}(x,\theta_t)} - 1\right)$$
$$= \frac{1}{Z_{\Lambda}}\sum_{J\subset\Lambda\setminus I}\sum_{y\in X_*^J} e^{\Delta_{\Lambda}\left(uy\theta_{t\cup(\Lambda\setminus I\setminus J)},\theta_{\Lambda}\right)}\left(e^{\Delta_t^{uy}(x,\theta_t)-\Delta_t^{u}(x,\theta_t)} - 1\right).$$

Using properties of Δ and condition (1), we can write

$$\Delta_t^{uy}(x,\theta_t) - \Delta_t^{u}(x,\theta_t)$$
$$= \sum_{j=1}^{|J|}\left(\Delta_{s_j}^{xuy_{s_{j+1}}\dots y_{s_{|J|}}}\left(y_{s_j},\theta_{s_j}\right) - \Delta_{s_j}^{uy_{s_{j+1}}\dots y_{s_{|J|}}}\left(y_{s_j},\theta_{s_j}\right)\right)$$
$$= \sum_{j=1}^{|J|}\left(\Delta_{s_j}^{xu}\left(y_{s_j},\theta_{s_j}\right) - \Delta_{s_j}^{u}\left(y_{s_j},\theta_{s_j}\right)\right),$$

where $J = \{s_1, s_2, \dots, s_{|J|}\}$ is some enumeration of points in J. Applying standard method (see, for example, [2, 7, 9, 10]), we obtain the expression for $G_\Lambda(xu)$ given in the statement of the theorem, which leads to equation

$$\rho_\Lambda(xu) = e^{\Delta_t^u(x,\theta_t)}\left(\rho_\Lambda(u) - \sum_{\alpha\in X_*^t}\rho_\Lambda(\alpha u) + G_\Lambda(xu)\right).$$

It remains to take the sum of the obtained relation over all $x \in X_*^t$ to get the expression for $\sum_{\alpha\in X_*^t}\rho_\Lambda(\alpha u)$ (see [8]).

□

4. Equations for correlation functions. Let $\Delta_1 = \{\Delta_t^{\bar{x}}, \bar{x} \in X^{t^c}, t \in \mathbb{Z}^d\}$ be a one-point transition energy field. We introduce the norm for Δ_1 as follows:

$$\|\Delta_1\| = \sup_{t\in\mathbb{Z}^d}\sup_{x\in X^t}\sum_{s\in t^c}\sup_{y\in X^s}|\Delta_s^x(y,\theta_s) - \Delta_s(y,\theta_s)|.$$

Put also

$$D = \sup_{t\in\mathbb{Z}^d}\sup_{x,u\in X^t}\sup_{\bar{x}\in X^{t^c}}\left|\Delta_t^{\bar{x}}(x,u)\right|.$$

We assume that $\mathbb{Z}^d$ is endowed with some order $\preccurlyeq$, for example, the lexicographical order. For each $I \in W$, denote $I' = I\setminus t$ where t is the smallest element in I with respect to $\preccurlyeq$. For the sake of simplicity, for any $x \in X_*^I$, we will use the notation $x' = x_{I'}$.

Consider the Banach space $\mathcal{B}_*$ of bounded functions φ on L_* with the norm

$$\|\varphi\| = \sup_{\Lambda\in W}\|\varphi\|_\Lambda, \qquad \|\varphi\|_\Lambda = \sum_{x\in X_*^\Lambda}|\varphi(x)|.$$

Consider the operator $\mathcal{K} = \mathcal{K}(\Delta_1)$ on $\mathcal{B}_*$ defined as follows:

$$(\mathcal{K}\varphi)(x) = \gamma(x)\big((S\varphi)(x) + (T\varphi)(x)\big), \qquad x \in X_*^I, I \in W,$$

where

$$\gamma(x) = \frac{e^{\Delta_t^u(x,\theta_t)}}{1 + \sum_{\alpha\in X_*^t} e^{\Delta_t^u(\alpha,\theta_t)}}, \qquad (S\varphi)(x) = \begin{cases}\varphi(x'), & |I| > 1,\\ 0, & |I| = 1,\end{cases}$$

and

$$(T\varphi)(x) = \sum_{\alpha\in X_*^t} e^{\Delta_t^u(\alpha,\theta_t)}\big((G\varphi)(x) - (G\varphi)(\alpha x')\big)$$

with

$$(G\varphi)(x) = \sum_{J\subset W(I^c)}\sum_{y\in X_*^J} K_{I\cup J}(xy)\left(\varphi(x'y) + \sum_{\alpha\in X_*^t}\varphi(\alpha x' y)\right)$$

and $K_{I\cup J}(xy)$ defined as in Theorem 2.

Further, we put

$$\delta(x) = \begin{cases}\gamma(x), & |I| = 1,\\ 0, & |I| > 1,\end{cases} \qquad x \in X_*^I, I \in W,$$

and

$$C_1 = \frac{e^D N_X}{1 + e^{-D} N_X}, \qquad C_2 = 4e^D N_X \left(exp\left\{e^{\|\Delta_1\|} - 1\right\} - 1\right), \qquad N_X = |X| - 1.$$

Using standard approach (see, for example, [7]), we obtain the following result.

Theorem 3. *Let one-point transition energy field Δ_1 be such that*

$$C_1(1 + C_2) < 1. \qquad (2)$$

Then the equation

$$\rho = \delta + \mathcal{K}\rho \qquad (3)$$

has a unique solution on $\mathcal{B}_*$ given by $\rho = \delta + \sum_{n=1}^{\infty} \mathcal{K}^n \delta$.

Now, for any $\Lambda \in W$, consider the operator $\mathcal{K}_\Lambda = \psi_\Lambda \mathcal{K} \psi_\Lambda$ where

$$(\psi_\Lambda \varphi)(x) = \begin{cases} \varphi(x), & x \in L_*^\Lambda, \\ 0, & \text{otherwise.} \end{cases}$$

According to Theorem 2, for each $\Lambda \in W$, the corresponding element ρ_Λ of $\{\rho_\Lambda, \Lambda \in W\}$ satisfies the equation

$$\rho_\Lambda = \delta_\Lambda + \mathcal{K}_\Lambda \rho_\Lambda \qquad (4)$$

where $\delta_\Lambda = \psi_\Lambda \delta$. Since $\|\mathcal{K}_\Lambda\| \le \|\mathcal{K}\|$, under conditions of Theorem 3 we have $\|\mathcal{K}_\Lambda\| \le 1$, and hence, ρ_Λ is the unique solution to (4), which can be written as $\rho_\Lambda = \delta_\Lambda + \sum_{n=1}^{\infty} \mathcal{K}_\Lambda^n \delta_\Lambda$.

The following theorem is the main result of the paper.

Theorem 4. *Let Δ_1 be a one-point transition energy field whose elements satisfy (1) and (2). Suppose also that there exists $R > 0$ such that for any $t \in \mathbb{Z}^d$,*

$$\Delta_t^{\bar{x}}(x, u) = \Delta_t^{\bar{x}_{\partial t}}(x, u), \qquad x, u \in X^t, \bar{x} \in X^{t^c}, \qquad (5)$$

where $\partial t = \{s \in t^c : |t - s| \le R\}$. Let $\{\rho_\Lambda, \Lambda \in W\}$ be the set of finite-volume correlation functions corresponding to Δ_1. Then for any $I \in W$,

$$\lim_{\Lambda \uparrow \mathbb{Z}^d} \rho_\Lambda(x) = \rho(x), \qquad x \in X_*^I,$$

where ρ is the solution of equation (3).

Remark. Let Φ be a pair interaction potential. Then the set $\Delta_1 = \left\{\Delta_t^{\bar{x}}, \bar{x} \in X^{t^c}, t \in \mathbb{Z}^d\right\}$ of functions

$$\Delta_t^{\bar{x}}(x, u) = \sum_{s \in t^c} \left(\Phi_{ts}(u\bar{x}_s) - \Phi_{ts}(x\bar{x}_s)\right), \qquad x, u \in X^t,$$

forms a one-point transition energy field corresponding to the potential Φ. It is not difficult to see that the elements of Δ_1 satisfy (1).

In particular, if Φ is a vacuum potential with the norm

$$\|\Phi\| = \sup_{t\in\mathbb{Z}^d} \sup_{x\in X_*^t} \sum_{s\in t^c} \sup_{y\in X_*^s} |\Phi_{ts}(xy)|,$$

then

$$\|\Delta_1\| \leq \|\Phi\|, \quad D \leq 2\|\Phi\|,$$

and condition (5) is satisfied if Φ is the finite-range potential. Thus, Theorem 4 holds true if

$$\frac{e^{2\|\Phi\|}N_X}{1 + e^{-2\|\Phi\|}N_X}\left(1 + 4e^{2\|\Phi\|}N_X\left(exp\{e^{\|\Phi\|} - 1\} - 1\right)\right) < 1.$$

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Transition Energy Field and Correlation Equations

Using the concept of the transition energy field, a system of correlation equations is obtained for lattice systems with finite spin space. It is shown that for a sufficiently small value of the one-point transition energies, the corresponding system of correlation functions, considered in infinite space, has a solution which is unique.

ՀՀ ԳԱԱ թղթակից անդամ Բ.Ս.Նահապետյան, Լ.Ա.Խաչատրյան

Անցումային էներգիայի դաշտ և կոռելյացիոն հավասարումներ

Օգտագործելով անցումային էներգիայի դաշտի գաղափարը՝ առաջարկվել է կոռելյացիոն հավասարումների ընդհանուր համակարգը: Ցույց է տրված, որ մեկկետանոց անցումային էներգիայի բավականին փոքր արժեքների համար այս համակարգը՝ դիտարկված անսահման տարածության մեջ, լուծելի է և ունի միակ լուծում:

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Поле энергий перехода и корреляционные уравнения

На основе понятия поля энергии перехода получена система корреляционных уравнений для решетчатых систем с конечным пространством спинов. Показано, что при достаточно малом значении одноточечных энергий перехода соответствующая система корреляционных функций, рассматриваемая в бесконечном пространстве, имеет решение, причем единственное.

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