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Derivatives of an Evolution Matrix and Lyapunov Exponents with Respect to Parameters

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Keywords: *nonlinear system, dependence of parameters, evolution matrix, Lyapunov exponents.*

1. Derivatives of an evolution matrix with respect to parameters. We consider a nonlinear dynamical system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{p},t), \quad \mathbf{x}(t_0) = \mathbf{x}_0 \tag{1.1}$$

where $\mathbf{f}$ is a smooth vector-function of the phase vector $\mathbf{x} \in R^m$, the vector of parameters $\mathbf{p} \in R^n$ and the time t, $\mathbf{x}_0$ is a given vector of initial conditions, and the dot means derivative with respect to time. We consider a point $\mathbf{p}$ in parameter space and take a small increment Δp_j to j-th component of the vector of parameters. Then the phase vector takes an increment $\Delta\mathbf{x}$ which in the first approximation is described by the differential equation with initial conditions

$$(\Delta\mathbf{x})^{\cdot} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{x},\mathbf{p},t)\Delta\mathbf{x} + \frac{\partial \mathbf{f}}{\partial p_j}(\mathbf{x},\mathbf{p},t)\Delta p_j, \quad \Delta\mathbf{x}(t_0) = 0 \tag{1.2}$$

where notation for the matrix

$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \left[\frac{\partial f_i}{\partial x_k}\right] \tag{1.3}$$

is used. Dividing both sides of equation (1.2) by Δp_j and taking the limit we obtain a differential equation with respect to the vector $\mathbf{z} = \partial\mathbf{x}/\partial p_j$ [1]

$$\dot{\mathbf{z}} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{x},\mathbf{p},t)\mathbf{z} + \frac{\partial \mathbf{f}}{\partial p_j}(\mathbf{x},\mathbf{p},t), \quad \mathbf{z}(t_0) = 0. \tag{1.4}$$

Consider now equation (1.1), linearized with respect to $\mathbf{x}$, for the vector $\mathbf{y} \in R^m$

$$\dot{\mathbf{y}} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{x},\mathbf{p},t)\mathbf{y} \tag{1.5}$$

A fundamental matrix for this equation (matriciant) is described by the initial value problem

$$\dot{\mathbf{Y}} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{x},\mathbf{p},t)\mathbf{Y}, \quad \mathbf{Y}(t_0) = \mathbf{I} \tag{1.6}$$

where $\mathbf{I}$ is the unit matrix. The matrix $\mathbf{Y}(t)$ is also called the evolution matrix [1, 2]. With this matrix solution to equation (1.5) with the initial condition $\mathbf{y}(t_0) = \mathbf{y}_0$ takes the form $\mathbf{y}(t) = \mathbf{Y}(t)\mathbf{y}_0$, and solution to equation (1.4) with zero initial condition is given by the integral [1]

$$\mathbf{z}(t) = \mathbf{Y}(t)\int_{t_0}^{t} \mathbf{Y}^{-1}(\tau)\frac{\partial \mathbf{f}}{\partial p_j}(\mathbf{x},\mathbf{p},\tau)d\tau \tag{1.7}$$

Now we find derivatives of the evolution matrix with respect to parameters at the point $\mathbf{p}$. Due to the increment Δp_j the matrix of evolution takes an increment $\mathbf{Y} + \Delta\mathbf{Y}$. Substituting this expression to equation (1.6) and expanding the right hand side in Taylor series we obtain an equation for the first approximation

$$\Delta\dot{\mathbf{Y}} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{x},\mathbf{p},t)\Delta\mathbf{Y} + \mathbf{C}\mathbf{Y}, \quad \Delta\mathbf{Y}(t_0) = \mathbf{0} \tag{1.8}$$

where the matrix $\mathbf{C}$ is given by the expression

$$\mathbf{C} = \frac{\partial^2 \mathbf{f}}{\partial \mathbf{x}^2}(\mathbf{x},\mathbf{p},t)\Delta\mathbf{x} + \frac{\partial^2 \mathbf{f}}{\partial \mathbf{x}\partial p_j}(\mathbf{x},\mathbf{p},t)\Delta p_j \tag{1.9}$$

Since $\mathbf{Y}(t)$ is the fundamental matrix, it is non-singular. We multiply both sides of equation (1.8) by the matrix $\mathbf{Y}^{-1}$ and integrate from t_0 to t. Using integration by parts with the initial condition (1.8) we obtain

$$\int_{t_0}^{t} \mathbf{Y}^{-1}\Delta\dot{\mathbf{Y}}d\tau = \mathbf{Y}^{-1}(t)\Delta\mathbf{Y}(t) - \int_{t_0}^{t}\left(\mathbf{Y}^{-1}\right)^{\cdot}\Delta\mathbf{Y}d\tau = \int_{t_0}^{t}\left(\mathbf{Y}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\Delta\mathbf{Y} + \mathbf{Y}^{-1}\mathbf{C}\mathbf{Y}\right)d\tau \tag{1.10}$$

To find the derivative $\left(\mathbf{Y}^{-1}\right)^{\cdot}$ we differentiate the identity $\mathbf{Y}^{-1}\mathbf{Y} = \mathbf{I}$ with respect to time and obtain $\left(\mathbf{Y}^{-1}\right)^{\cdot}\mathbf{Y} + \mathbf{Y}^{-1}\dot{\mathbf{Y}} = 0$. Thus, $\left(\mathbf{Y}^{-1}\right)^{\cdot} = -\mathbf{Y}^{-1}\dot{\mathbf{Y}}\mathbf{Y}^{-1}$ and with equation (1.6) we find

$$\left(\mathbf{Y}^{-1}\right)^{\cdot} = -\mathbf{Y}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \tag{1.11}$$

We use expression (1.11) in the second equality (1.10) and obtain

$$\Delta\mathbf{Y}(t) = \mathbf{Y}(t)\int_{t_0}^{t}\mathbf{Y}^{-1}\mathbf{C}\mathbf{Y}d\tau \tag{1.12}$$

We substitute here relation (1.9) for the matrix $\mathbf{C}$ and divide both sides of the equality by Δp_j. As a result, we get

$$\cdot\frac{\Delta \mathbf{Y}(t)}{\Delta p_j} = \mathbf{Y}(t)\int_{t_0}^{t} \mathbf{Y}^{-1}\left(\frac{\partial^2 \mathbf{f}}{\partial \mathbf{x}^2}\frac{\Delta \mathbf{x}}{\Delta p_j} + \frac{\partial^2 \mathbf{f}}{\partial \mathbf{x}\partial p_j}\right)\mathbf{Y}\,d\tau \tag{1.13}$$

Taking in this equation the limit and using notation $\mathbf{z} = \partial\mathbf{x}/\partial p_j$ we obtain

$$\frac{\partial \mathbf{Y}(t)}{\partial p_j} = \mathbf{Y}(t)\int_{t_0}^{t} \mathbf{Y}^{-1}\left(\frac{\partial^2 \mathbf{f}}{\partial \mathbf{x}^2}\mathbf{z} + \frac{\partial^2 \mathbf{f}}{\partial \mathbf{x}\partial p_j}\right)\mathbf{Y}\,d\tau \tag{1.14}$$

Substituting here the vector $\mathbf{z}$ from (1.7) we finally obtain the expression for the derivative of the evolution matrix with respect to parameters as

$$\frac{\partial \mathbf{Y}(t)}{\partial p_j} = \mathbf{Y}(t)\int_{t_0}^{t} \mathbf{Y}^{-1}(\tau)\left(\frac{\partial^2 \mathbf{f}}{\partial \mathbf{x}^2}\mathbf{Y}(\tau)\int_{t_0}^{\tau}\mathbf{Y}^{-1}(\varsigma)\frac{\partial \mathbf{f}}{\partial p_j}(\mathbf{x},\mathbf{p},\varsigma)d\varsigma + \frac{\partial^2 \mathbf{f}}{\partial \mathbf{x}\partial p_j}(\mathbf{x},\mathbf{p},\tau)\right)\mathbf{Y}(\tau)\,d\tau \tag{1.15}$$

Hence, to calculate the evolution matrix $\mathbf{Y}(t)$ and its derivatives with respect to parameters we need to integrate equations (1.1) and (1.6) with the corresponding initial conditions and evaluate the integrals (1.15). It is easy to see that in this case it is necessary to integrate $m(1+m)$ differential equations of the first order, and in contrast, if we calculate the first order derivatives numerically it is necessary to integrate at least $m(1+m)(1+n)$ equations of the first order. Thus, the difference in the number of equations to be solved is $mn(1+m)$, and it is as higher as many degrees of freedom and problem parameters are involved. Knowing derivatives of the evolution matrix allows to predict behavior of a dynamical system in the vicinity of the point $\mathbf{p}$ in parameter space.

2. Periodic solutions. We consider the case when equation (1.1) possesses a periodic solution $\mathbf{x}(t) = \mathbf{x}(t+T)$ and the matrix $\partial\mathbf{f}(\mathbf{x}(t),\mathbf{p},t)/\partial\mathbf{x}$ is periodic with a period T.

Then the evolution matrix $\mathbf{F} = \mathbf{Y}(T)$ is called the monodromy matrix [2-4]. According to the Floquet theory, eigenvalues of this matrix (multipliers) are responsible for the stability of the periodic solution $\mathbf{x}(t)$: if all the multipliers by their absolute value are less than unity, then the periodic solution $\mathbf{x}(t)$ is asymptotically stable, and it is unstable if at least one of the multipliers is greater than unity by its absolute value [2-4].

In particular, if we consider a linear system $\dot{\mathbf{x}} = \mathbf{G}(\mathbf{p},t)\mathbf{x}$ with the periodic matrix $\mathbf{G}(\mathbf{p},t+T) = \mathbf{G}(\mathbf{p},t)$ and study the stability of the trivial solution $\mathbf{x}(t) \equiv 0$, then we have

$$\mathbf{f} = \mathbf{G}\mathbf{x}, \quad \frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \mathbf{G}, \quad \frac{\partial^2 \mathbf{f}}{\partial \mathbf{x}^2} = \mathbf{0}, \quad \frac{\partial^2 \mathbf{f}}{\partial \mathbf{x}\partial p_j} = \frac{\partial \mathbf{G}}{\partial p_j} \tag{2.1}$$

Therefore, for this case according to (1.15) we get

$$\frac{\partial \mathbf{Y}(t)}{\partial p_j} = \mathbf{Y}(t)\int_{t_0}^{t} \mathbf{Y}^{-1}\frac{\partial \mathbf{G}}{\partial p_j}\mathbf{Y}\,d\tau \tag{2.2}$$

From this formula at $t = T$ we obtain the derivatives of the monodromy matrix $\mathbf{F} = \mathbf{Y}(T)$ with respect to parameters [5]

$$\frac{\partial \mathbf{F}}{\partial p_j} = \mathbf{F}\int_{t_0}^{T} \mathbf{Y}^{-1} \frac{\partial \mathbf{G}}{\partial p_j} \mathbf{Y} d\tau \tag{2.3}$$

Note that to find derivatives (2.2) and (2.3) it is necessary to know only the matriciant $\mathbf{Y}(t)$ and the derivatives of the matrix $\mathbf{G}$ with respect to parameters taken at the point $\mathbf{p}$. Using derivatives (2.3) a variation of the monodromy matrix can be given in the form

$$\mathbf{F}(\mathbf{p} + \Delta\mathbf{p}) = \mathbf{F} + \sum_{k=1}^{n} \frac{\partial \mathbf{F}}{\partial p_k} \Delta p_k + \ldots \tag{2.4}$$

Knowing the derivatives of the monodromy matrix we can calculate the value of this matrix in the vicinity of the initial point $\mathbf{p}$, and therefore estimate behavior of the multipliers responsible for the stability of the system when the problem parameters are changed. The second and higher order derivatives of the monodromy matrix were derived in [5, 6].

As an example in this subsection we consider Hill's equation with damping

$$\ddot{x} + \beta\dot{x} + [\omega^2 + \varepsilon\varphi(t)]\, x = 0 \tag{2.5}$$

where β is the damping coefficient, ε is the excitation amplitude, ω is the natural frequency, and $\varphi(t)$ is a continuous periodic function of time with the period 2π having zero mean value $\int_0^{2\pi} \varphi(t)dt = 0$. For the initial point we take $t_0 = 0$. Equation (2.5) has been considered by many authors.

Our aim is to find analytically the domains of instability of the trivial solution $x = 0$ (the domains of parametric resonance) in the case of small excitation amplitude ε, damping coefficient β, and arbitrary natural frequency $\omega \neq 0$. For this purpose we represent equation (2.5) in the form (1.5)

$$\dot{\mathbf{y}} = \mathbf{G}(\mathbf{p}, t)\mathbf{y} \tag{2.6}$$

with

$$\mathbf{y} = \begin{pmatrix} x \\ \dot{x} \end{pmatrix}, \quad \mathbf{G}(t, \mathbf{p}) = \begin{bmatrix} 0 & 1 \\ -\omega^2 - \varepsilon\varphi(t) & -\beta \end{bmatrix} \tag{2.7}$$

This system contains three parameters $\mathbf{p} = (\varepsilon, \beta, \omega)$. If we substitute in (2.7) then it is easy to find from (1.6) and (2.6), (2.7) the matriciant and its inverse as

$$\mathbf{Y}(t)=\begin{bmatrix}\cos\omega t & \omega^{-1}\sin\omega t\\ -\omega\sin\omega t & \cos\omega t\end{bmatrix},\ \mathbf{Y}^{-1}(t)=\begin{bmatrix}\cos\omega t & -\omega^{-1}\sin\omega t\\ \omega\sin\omega t & \cos\omega t\end{bmatrix} \quad (2.8)$$

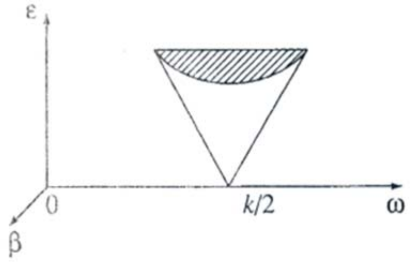

Fig. 1. The instability domains for Hill's equation with damping.

So, when $\varepsilon = 0, \beta = 0$ we have the monodromy matrix in the form

$$\mathbf{F}=\mathbf{Y}(2\pi)=\begin{bmatrix}\cos 2\pi\omega & \omega^{-1}\sin 2\pi\omega\\ -\omega\sin 2\pi\omega & \cos 2\pi\omega\end{bmatrix} \quad (2.9)$$

The eigenvalues of this matrix (the multipliers) are

$$\rho_{1,2}=\cos 2\pi\omega \pm i\sin 2\pi\omega \quad (2.10)$$

If $\omega \neq k/2, k=1,2,...$ the multipliers are complex conjugate quantities lying on the unit circle (stability). We can show that when small periodic excitation and damping are added $(\varepsilon > 0, \beta > 0)$ then the multipliers move inside the unit circle which implies asymptotic stability. Indeed, in this case the multipliers due to continuity property remain complex conjugate quantities. For the multipliers we have a quadratic equation

$$\rho^2 + c_1\rho + c_2 = 0 \quad (2.11)$$

According to Vieta's theorem and Liouville's formula [1,3] the last coefficient c_2 is equal to

$$c_2=\rho_1\rho_2=|\rho|^2=\exp\left(\int_0^{2\pi} tr(\mathbf{G})dt\right)=\exp(-2\pi\beta)<1 \quad (2.12)$$

This inequality means that when small periodic excitation and damping are added to the unperturbed system the complex conjugate multipliers move inside the unit circle, and the system becomes asymptotically stable.

Therefore, the instability (parametric resonance) can take place only in the vicinity of the points

$$\mathbf{p}_0: \ \varepsilon=0, \beta=0, \omega=k/2, \ k=1,2,... \quad (2.13)$$

at which the multipliers are double $\rho_1=\rho_2=(-1)^k$.

To find the instability domains we expand the monodromy matrix $\mathbf{F}$ in Taylor's series in the vicinity of the points $\mathbf{p}_0$ with respect to the parameters ε, β, and $\Delta\omega = \omega - k/2$

$$\mathbf{F}(\mathbf{p}) = \mathbf{F}(\mathbf{p}_0) + \frac{\partial \mathbf{F}}{\partial \varepsilon}\varepsilon + \frac{\partial \mathbf{F}}{\partial \beta}\beta + \frac{\partial \mathbf{F}}{\partial \omega}\Delta\omega + \cdots \tag{2.14}$$

According to formula (2.3) and with the use of (2.6)-(2.8) we calculate the derivatives $\partial\mathbf{F}/\partial\varepsilon$, $\partial\mathbf{F}/\partial\beta$, and $\partial\mathbf{F}/\partial\omega$ at $\mathbf{p} = \mathbf{p}_0$. Then, up to the first order terms (2.14) we get

$$\mathbf{F}(\mathbf{p}) = \cos \pi k \begin{bmatrix} 1 + \pi a_k \varepsilon / k - \pi\beta & 2\pi(2\Delta\omega k - b_k \varepsilon)/k^2 \\ -\pi(\Delta\omega k + b_k \varepsilon / 2) & 1 - \pi a_k \varepsilon / k - \pi\beta \end{bmatrix} \tag{2.15}$$

Here we have introduced notation for the Fourier coefficients of the periodic function $\varphi(t)$

$$a_k = \frac{1}{\pi}\int_0^{2\pi} \varphi(t) \sin ktdt, \qquad b_k = \frac{1}{\pi}\int_0^{2\pi} \varphi(t) \cos ktdt, \quad k = 1,2,... \tag{2.16}$$

The eigenvalues of matrix (2.15) (the multipliers) can be found approximately as

$$\rho_{1,2} = (-1)^k (1 - \pi\beta) \pm \pi\sqrt{D}, \tag{2.17}$$

$$D = (a_k^2 + b_k^2)\varepsilon^2 / k^2 - 4(\Delta\omega)^2 \tag{2.18}$$

The system is unstable if the absolute value of at least one multiplier is greater than one. This condition is fulfilled for $\beta < 0$, and the system becomes unstable. But if $\beta \geq 0$ this instability condition is satisfied only when $\sqrt{D} > \beta$. Hence, using (2.18) we obtain the instability (parametric resonance) domain as

$$4(\omega - k/2)^2 + \beta^2 < (a_k^2 + b_k^2)\varepsilon^2 / k^2, \ \beta \geq 0 \tag{2.19}$$

It is half of the cone in three-parameter space joining the half-space $\beta < 0$, Fig 1.

Formula (2.19) agrees with those obtained earlier for some specific cases. For instance, if we put in (2.19) $\beta = 0, \varphi(t) = \cos t$ we get the Mathieu equation. In this case for the domain of the first resonance $k = 1$ we get according to (2.16) $a_1 = 0,\ b_1 = 1$, and from (2.19) we obtain the well known relation $1 - \varepsilon < 2\omega < 1 + \varepsilon$, see [2, 4]. Thus, for periodic systems it is shown how derivatives of the evolution matrix provide the instability domains.

3. The Lyapunov exponents. Consider now an autonomous system (1.1)

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{p}), \qquad \mathbf{x}(t_0) = \mathbf{x}_0 \tag{3.1}$$

The evolution matrix $\mathbf{Y}(t)$ is found from a linearized equation

$$\dot{\mathbf{Y}} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{x},\mathbf{p})\mathbf{Y}, \quad \mathbf{Y}(t_0) = \mathbf{I} \tag{3.2}$$

In [7, 8] the Lyapunov exponents are defined by means of an eigenvalue problem for the matrix $\mathbf{V}_T = \mathbf{Y}_T^+ \mathbf{Y}_T$, where cross means transposition, and T is an upper boundary for the time integration

$$\mathbf{V}_T \mathbf{u} = \kappa \mathbf{u} \tag{3.3}$$

The matrix $\mathbf{V}_T$ is symmetric and positive definite. Indeed, $\left(\mathbf{Y}_T^+ \mathbf{Y}_T\right)^+ = \mathbf{Y}_T^+ \mathbf{Y}_T$ and for an arbitrary vector $\mathbf{y}$ the inequality $\mathbf{y}^+ \mathbf{Y}_T^+ \mathbf{Y}_T \mathbf{y} = (\mathbf{Y}_T \mathbf{y})^+ \mathbf{Y}_T \mathbf{y} = \left\| \mathbf{Y}_T \mathbf{y} \right\|^2 > 0$ is satisfied. The strict inequality takes place since $\mathbf{Y}_T$ is fundamental and thus, non-singular matrix. Hence, all the eigenvalues κ_s, $s = 1,\ldots,m$ of the matrix $\mathbf{V}_T$ are positive. The eigenvalues of the matrix $\sqrt{\mathbf{V}_T}$ are called singular values of the matrix $\mathbf{Y}_T$.

The Lyapunov exponents are defined as a limit of the ratio [7, 8]

$$\Lambda_s = \lim_{T \to \infty} \left(\frac{\ln \kappa_s}{2T} \right) \tag{3.4}$$

Let $\mathbf{u}_s$ be an eigenvector, corresponding to a simple eigenvalue κ_s of the matrix $\mathbf{V}_T$. Then, the derivative of the eigenvalue κ_s with respect to parameters is

$$\frac{\partial \kappa_s}{\partial p_j} = \mathbf{u}_s^+ \frac{\partial (\mathbf{Y}_T^+ \mathbf{Y}_T)}{\partial p_j} \mathbf{u}_s / (\mathbf{u}_s^+ \mathbf{u}_s) = \mathbf{u}_s^+ \left[\left(\frac{\partial \mathbf{Y}_T}{\partial p_j} \right)^+ \mathbf{Y}_T + \mathbf{Y}_T^+ \left(\frac{\partial \mathbf{Y}_T}{\partial p_j} \right) \right] \mathbf{u}_s / (\mathbf{u}_s^+ \mathbf{u}_s) \tag{3.5}$$

For the Lyapunov exponents we have

$$\frac{\partial \Lambda_s}{\partial p_j} = \lim_{T \to \infty} \frac{1}{2T} \frac{\partial \ln \kappa_s}{\partial p_j} = \lim_{T \to \infty} \frac{1}{2T\kappa_s} \frac{\partial \kappa_s}{\partial p_j}. \tag{3.6}$$

Substituting expression (3.5) in formula (3.6) we finally obtain

$$\frac{\partial \Lambda_s}{\partial p_j} = \lim_{T \to \infty} \mathbf{u}_s^+ \left[\left(\frac{\partial \mathbf{Y}_T}{\partial p_j} \right)^+ \mathbf{Y}_T + \mathbf{Y}_T^+ \left(\frac{\partial \mathbf{Y}_T}{\partial p_j} \right) \right] \mathbf{u}_s / \left(2T\kappa_s \mathbf{u}_s^+ \mathbf{u}_s \right) \tag{3.7}$$

This formula relates derivatives of the Lyapunov exponents with respect to parameters to the derivatives of the evolution matrix derived in (1.15).

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Derivatives of an Evolution Matrix and Lyapunov Exponents with Respect to Parameters

A nonlinear dynamical system dependent on parameters is considered. Formulas for derivatives of an evolution matrix of the system with respect to parameters are derived in the form of integrals of a function depending on the phase vector, its derivatives with respect to parameters and the evolution matrix taken at a given point in the parameter space. For autonomous systems formulas for derivatives of the Lyapunov exponents are derived and expressed through the derivatives of the evolution matrix with respect to parameters. The obtained formulas simplify analysis of behavior of the dynamical system under change of problem parameters.

ՀՀ ԳԱԱ արտասահմանյան անդամ Ա. Պ. Սեյրանյան

Էվոլյուցիայի մատրիցայի և Լյապունովի ցուցիչների ածանցյալները՝ ըստ պարամետրերի

Դիտարկված է պարամետրերից կախված ոչ գծային դինամիկ համակարգ: Համակարգի էվոլյուցիայի մատրիցայի համար դուրս են բերված նրա ըստ պարամետրերի ածանցյալները: Ստացված ածանցյալները պարամետրերի տարածության տրված կետում ներկայացված են ըստ ժամանակի ինտեգրալների տեսքով, վերցված ֆազային ֆունկցիայի վեկտորից, նրա ըստ պարամետրերի ածանցյալներից և էվոլյուցիայի մատրիցայից: Ստացված են Լյապունովի ցուցիչների ածանցյալների բանաձևերը ինքնավար համակարգերի համար, որոնք արտահայտված են էվոլյուցիայի մատրիցայի ըստ պարամետրերի ածանցյալներով: Ստացված բանաձևերը թույլ են տալիս պարզեցնել համակարգի դինամիկայի վարքի վերլուցությունը պարամետրերի փոփոխության ընթացքում:

Иностранный член НАН РА А. П. Сейранян

Производные матрицы эволюции и показателей Ляпунова по параметрам

Рассматривается нелинейная динамическая система, зависящая от параметров. Выведены формулы для производных матрицы эволюции системы по параметрам в виде интегралов по времени от функции фазового вектора, его производных по параметрам и матрицы эволюции системы в заданной точке пространства параметров. Для автономных систем найдены формулы для производных показателей Ляпунова, выраженные через производные матрицы эволюции по параметрам. Полученные формулы позволяют упростить анализ динамического поведения системы при изменении параметров.

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