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On Effective Implementation of Recognition Algorithms for Calculating Estimates

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1. Introduction. The statement of the recognition problem is given in [1]. Let $M = \bigcup_{i=1}^{l} K_i \subseteq M_1 \times M_2 \times \ldots \times M_n, n > 1$, where M_i is a metric space with a metric $d_i, i = 1,2,\ldots,n$, be the set of admissible objects. Subsets K_i are called classes and $K_i \neq \emptyset, i = 1,2,\ldots,l$.

For an admissible object S classification is done by calculating estimates $\Gamma_i(S)$ to the class K_i. Each algorithm in a model is determined by choosing a system of supporting sets, a proximity function, weights of the admissible objects, weights of the attributes, and a decision rule. The system of supporting sets Ω_A of an algorithm A is a nonempty set of subsets of $\{1,2,\ldots,n\}$. Each element $\Omega \in \Omega_A$ can be described by its characteristic vector $\omega_\Omega = (\omega_1, \omega_2, \ldots, \omega_n)$, where $\omega_\Omega^i = 1$ if and only if $i \in \Omega$. Denote $W_{\Omega_A} = \{\omega_\Omega \mid \Omega \in \Omega_A\}$.

In each algorithm A integers $q_1, q_2 \geq 0$ and $\varepsilon_i \geq 0, i = 1,2,\ldots,n$ are fixed. For admissible objects $S = (s_1, s_2, \ldots, s_n), S' = (s'_1, s'_2, \ldots, s'_n)$, and a supporting set Ω the proximity function $B(\Omega, S, S')$ is defined as

$$B(\Omega, S, S') = \begin{cases} 1, & (\delta \cdot \omega_\Omega) \geq q_1, (\overline{\delta} \cdot \omega_\Omega) \leq q_2 \\ 0, & otherwise \end{cases}$$

where $\delta = \delta(S, S') = (\delta_1, \delta_2, \ldots, \delta_n)$ such that

$$\delta_i = \begin{cases} 1, & d_i(s_i, s'_i) \leq \varepsilon_i \\ 0, & d_i(s_i, s'_i) > \varepsilon_i \end{cases}$$

Let $\gamma(S)$ be the weight of an admissible object S. Denote the weight of the attribute $i \in \{1,2,\ldots,n\}$ by $\mu_i \geq 0$. The weight of a supporting set $\Omega = \{i_1, i_2, \ldots, i_k\}$ is defined as $\mu(\omega_\Omega) = \mu_{i_1} + \mu_{i_2} + \cdots + \mu_{i_k}$. The estimate $\Gamma_i(S)$ is defined as

$$\Gamma_i(S)=\frac{1}{F|K_i|}\sum_{S'\in K_i}\gamma(S')\sum_{\Omega\in\Omega_A}\mu(\omega_\Omega)B(\Omega,S,S') \tag{1}$$

where F is a normalizing factor. Formula (1) is practically inefficient because the number of terms in the inner sum may be exponentially large. The following formula is proposed for the calculation of the estimates [2]:

$$\Gamma_i(S)=\frac{1}{F|K_i|}\sum_{S'\in K_i}\gamma(S')\sum_{i=1}^{n}\mu_i Q_i(S,S') \tag{2}$$

where $Q_i(S,S')=|\Omega\in\Omega_A\mid i\in\Omega,\ B(\Omega,S,S')=1|$. Now the number of summations in (2) is not greater than n. If the number of distinct values of $Q_i(S,S')$ is small, then the calculation of the inner sum almost disappears. Hence (2) is an effective formula for the estimates calculation.

By $\|\alpha\|$ we denote the number of ones in a binary vector α. For binary vectors α and β, $\alpha+\beta$ denotes the logical XOR operation.

Suppose $\delta_i=1$ if and only if $i\in\Delta=\{j_1,j_2,\dots,j_m\}, j_1<j_2<\dots<j_m$. For $\alpha=(\alpha_1,\alpha_2,\dots,\alpha_n)$ denote $\alpha^1=\left(\alpha_{j_1},\alpha_{j_2},\dots,\alpha_{j_m}\right)$ and $\alpha^2=(\alpha_{k_1},\alpha_{k_2},\dots,\alpha_{k_{n-m}})$, where $\{k_1,k_2,\dots,k_{n-m}\}=\{1,2,\dots,n\}\setminus\Delta$, $k_1<k_2<\dots<k_{n-m}$. Then

$$B(\Omega,S,S')\equiv B(\Omega,\delta,\Delta,q_1,q_2)=\begin{cases}1, & \|\delta^1+\omega_\Omega^1\|\le|\Delta|-q_1, \|\delta^2+\omega_\Omega^2\|\le q_2\\ 0, & otherwise\end{cases}$$

The latter defines $B(\Omega,\delta,\Delta,q_1,q_2)$ even when δ and Δ are not related, i.e. $B(\Omega,\delta,\Delta,q_1,q_2)$ is defined for $\forall\delta\in E^n,\forall\Omega,\Delta\subseteq\{1,2,\dots,n\}$, $\forall q_1,q_2\ge 0$. Unless otherwise stated, we assume that δ and Δ are not related.

Define $Q_i(\delta,\Delta,q_1,q_2)=|\Omega\in\Omega_A\mid i\in\Omega,\ B(\Omega,\delta,\Delta,q_1,q_2)=1|$, $\tilde{\imath}(\Omega_A)=\{\Omega\in\Omega_A\mid i\in\Omega\}$ and $\overline{Q}_i(\delta,\Delta,q_1,q_2)=|\tilde{\imath}(\Omega_A)|-Q_i(\delta,\Delta,q_1,q_2)$.

Definition 1.1. *A system Ω_A of supporting sets is said to have rank k, if* $|\{Q_i(\delta,\Delta,q_1,q_2)\}_{i=1}^n|\le k$, *for* $\forall\delta\in E^n,\forall\Delta\subseteq\{1,2,\dots,n\},\forall q_1,q_2\ge 0$, *and* $\left|\{Q_i(\delta^0,\Delta^0,q_1^0,q_2^0)\}_{i=1}^n\right|=k$ *for some* $(\delta^0,\Delta^0,q_1^0,q_2^0)$.

Definition 1.2. *A system Ω_A of supporting sets is said to have Δ-rank k, if* $|\{Q_i(\delta,\Delta,q_1,q_2)\}_{i=1}^n|\le k$, *for* $\forall\delta\in E^n,\Delta=\{i\mid\delta_i=1\},\forall q_1,q_2\ge 0$, *and* $\left|\{Q_i(\delta^0,\Delta^0,q_1^0,q_2^0)\}_{i=1}^n\right|=k$ *for some* $(\delta^0,\Delta^0,q_1^0,q_2^0)$, $\Delta^0=\{i\mid\delta_i^0=1\}$.

The rank and the Δ-rank of the system Ω_A are denoted by $R(\Omega_A)$ and $R_\Delta(\Omega_A)$ respectivally. Clearly $R_\Delta(\Omega_A)\le R(\Omega_A)$.

Definition 1.3. *A system Ω_A of supporting sets is called absolutely reducible if* $|\tilde{\imath}(\Omega_A)|\le 1,\forall i\in\{1,2,\dots,n\}$ *or* $|\tilde{\imath}(\Omega_A)|\ge n-1,\forall i\in\{1,2,\dots,n\}$.

Definition 1.4. *A system Ω_A of supporting sets is called absolutely symmetric if for each $\Omega\in\Omega_A$ it follows that* $E_n^{\|\omega_\Omega\|}\subseteq W_{\Omega_A}$.

Definition 1.5. *A system Ω_A of supporting sets is called internal if* $\varnothing\notin\Omega_A$ *and* $\{1,2,\dots,n\}\notin\Omega_A$.

Clearly, when considering the rank and the Δ-rank of a system Ω_A, without loss of generality we may assume that Ω_A is internal. The following theorem is due to [3]:

Theorem 1.6. *Given an internal system Ω_A of supporting sets, $R_\Delta(\Omega_A) \leq 2$ if and only if Ω_A is either absolutely reducible or absolutely symmetric.*

For the rest of the paper we will assume that Ω_A is internal. If Ω_A is absolutely symmetric then $R(\Omega_A) \leq 4$ [2]. Thus, for absolutely reducible and absolutely symmetric supporting sets we only have upper bounds for $R(\Omega_A)$ and $R_\Delta(\Omega_A)$. In section 2 we calculate the exact values of $R(\Omega_A)$ and $R_\Delta(\Omega_A)$ for these supporting sets. Finally, in section 3 we suggest a general method for constructing effective algorithms.

2. Ranks of absolutely reducible and absolutely symmetric supporting sets. Proposition 2.1. *If Ω_A is absolutely symmetric, then $R_\Delta(\Omega_A) = 2$.*

Proof. Let $\delta = (0,1,1,\ldots,1)$, $\Delta = \{2,3,\ldots,n\}$, $q_1 = q_2 = 0$. Then $Q_1(\delta,\Delta,q_1,q_2) = 0$ and $Q_i(\delta,\Delta,q_1,q_2) \neq 0, i \neq 1$ and $R_\Delta(\Omega_A) = 2$.

Proposition 2.2. *If Ω_A is absolutely reducible, then $R(\Omega_A) = R_\Delta(\Omega_A) = 2$.*

Proof. If $|\tilde{\imath}(\Omega_A)| \neq |\tilde{\jmath}(\Omega_A)|$ for some $i,j \in \{1,2,\ldots,n\}, i \neq j$, then taking $\forall\delta \in E^n, q_1 = 0, q_2 = n$ we have $R_\Delta(\Omega_A) = 2$. Otherwise, we take the same approach as in the proof of proposition 2.1.

It is easy to see that $R(\Omega_A) \leq 2$. Thus we also have $R(\Omega_A) = 2$.

To calculate the ranks of absolutely symmetric supporting sets we need a few lemmas. To avoid considering various trivial cases we will assume that $n > 5$.

Lemma 2.3. *For $W_{\Omega_A} = E_n^2$ we have $R(\Omega_A) = 4$.*

Proof. Let $\delta = (0,0,0,1,1,\ldots,1), \Delta = \{2,3,\ldots,n-1\}, q_1 = 1, q_2 = 1$. Then $\delta^1 = (0,0,1,\ldots,1), \delta^2 = (0,1)$ and

$$B(\Omega,\delta,\Delta,q_1,q_2) = \begin{cases} 1, & \|\delta^1 + \omega_\Omega^1\| \leq n-3, \|\delta^2 + \omega_\Omega^2\| \leq 1 \\ 0, & otherwise \end{cases}$$

Since $\delta^1 \in E^{n-2}, \|\delta^1 + \omega_\Omega^1\| > n-3$ means $\omega_\Omega^1 = \overline{\delta^1} = (1,1,0,\ldots,0)$. As $\omega_\Omega \in E_n^2$, the latter implies $\omega_\Omega = (0,1,1,0,\ldots,0)$. Now

$$\left|\left\{\Omega \in \Omega_A \mid 1 \in \Omega, \|\delta^2 + \omega_\Omega^2\| > 1\right\}\right| = \binom{n-2}{1} = n-2$$

and

$$\left|\left\{\Omega \in \Omega_A \mid 1 \in \Omega, \|\delta^1 + \omega_\Omega^1\| > n-3, \|\delta^2 + \omega_\Omega^2\| \leq 1\right\}\right| = 0$$

Hence $\bar{Q}_1(\delta,\Delta,q_1,q_2) = n-2$. For $\bar{Q}_2(\delta,\Delta,q_1,q_2)$ we have

$$\left|\left\{\Omega \in \Omega_A \mid 2 \in \Omega, \|\delta^2 + \omega_\Omega^2\| > 1\right\}\right| = 1$$

and

$$\left|\left\{\Omega \in \Omega_A \mid 2 \in \Omega, \|\delta^1 + \omega_\Omega^1\| > n-3, \|\delta^2 + \omega_\Omega^2\| \leq 1\right\}\right| = 1$$

Therefore, $\bar{Q}_2(\delta,\Delta,q_1,q_2) = 2$. Similarly, we get $\bar{Q}_4(\delta,\Delta,q_1,q_2) = 1$ and $\bar{Q}_n(\delta,\Delta,q_1,q_2) = 0$. From

$$\bar{Q}_1(\delta,\Delta,q_1,q_2) > \bar{Q}_2(\delta,\Delta,q_1,q_2) > \bar{Q}_4(\delta,\Delta,q_1,q_2) > \bar{Q}_n(\delta,\Delta,q_1,q_2)$$

it follows that

$$Q_1(\delta,\Delta,q_1,q_2) < Q_2(\delta,\Delta,q_1,q_2) < Q_4(\delta,\Delta,q_1,q_2) < Q_n(\delta,\Delta,q_1,q_2)$$

and $R(\Omega_A) = 4$.

Lemma 2.4. *For $W_{\Omega_A} = E_n^3$ we have $R(\Omega_A) = 4$.*

Proof. Let $\delta = (0,0,0,1,1,\dots,1), \Delta = \{2,3,\dots,n-1\}, q_1 = 1, q_2 = 1$. In the same way as in lemma 2.3, we can show that

$$\bar{Q}_1(\delta,\Delta,q_1,q_2) = \binom{n-2}{2}, \bar{Q}_2(\delta,\Delta,q_1,q_2) = n-2$$
$$\bar{Q}_4(\delta,\Delta,q_1,q_2) = n-3, \bar{Q}_n(\delta,\Delta,q_1,q_2) = 1$$

Hence $Q_1(\delta,\Delta,q_1,q_2) < Q_2(\delta,\Delta,q_1,q_2) < Q_4(\delta,\Delta,q_1,q_2) < Q_n(\delta,\Delta,q_1,q_2)$ and $R(\Omega_A) = 4$.

Corollary 2.5. *If* $W_{\Omega_A} = E_n^2$ *or* $W_{\Omega_A} = E_n^3$ *then for* $\delta = (0,0,0,1,1,\dots,1)$ $, \mathit{\Delta} = \{2,3,\dots,n-1\}, q_1 = q_2 = 1$ *we have* $Q_1(\delta,\mathit{\Delta},q_1,q_2) < Q_2(\delta,\mathit{\Delta},q_1,q_2)$ $< Q_4(\delta,\mathit{\Delta},q_1,q_2) < Q_n(\delta,\mathit{\Delta},q_1,q_2)$.

Lemma 2.6. *For* $W_{\Omega_A} = E_n^k, 3 < k < n-1$ *we have* $R(\Omega_A) = 4$.

Proof. Let $\delta = (0,0,0,1,1,\dots,1), \Delta = \{2,3,\dots,n-1\}, q_1 = k-3, q_2 = 1$. Then $\delta^1 = (0,0,1,\dots,1), \delta^2 = (0,1)$ and

$$B(\Omega,\delta,\Delta,q_1,q_2) = \begin{cases} 1, & \|\delta^1 + \omega_\Omega^1\| \le n-k+1, \|\delta^2 + \omega_\Omega^2\| \le 1 \\ 0, & otherwise \end{cases}$$

The value of $\binom{m}{l}$, where $l < 0$, is considered to be zero.

Again, by considering $\left|\{\Omega \in \Omega_A \mid i \in \Omega, \|\delta^2 + \omega_\Omega^2\| > 1\}\right|$ and $\left|\{\Omega \in \Omega_A \mid i \in \Omega, \|\delta^1 + \omega_\Omega^1\| > n-k+1, \|\delta^2 + \omega_\Omega^2\| \le 1\}\right| = 0$ for $i = 1,2,4$, n, we get

$$\bar{Q}_1(\delta,\Delta,q_1,q_2) = \binom{n-2}{k-1} + \binom{n-4}{k-4}, \bar{Q}_2(\delta,\Delta,q_1,q_2) = \binom{n-3}{k-2} + \binom{n-4}{k-4},$$
$$\bar{Q}_4(\delta,\Delta,q_1,q_2) = \binom{n-3}{k-2} + \binom{n-5}{k-5}, \bar{Q}_n(\delta,\Delta,q_1,q_2) = \binom{n-4}{k-4}$$

Hence,

$$Q_1(\delta,\Delta,q_1,q_2) < Q_2(\delta,\Delta,q_1,q_2) < Q_4(\delta,\Delta,q_1,q_2) < Q_n(\delta,\Delta,q_1,q_2)$$

and $R(\Omega_A) = 4$.

Corollary 2.7. *Let* $W_{\Omega_A} = E_n^k, \delta = (0,0,0,1,1,\dots,1), \mathit{\Delta} = \{2,3,\dots,n-1\}$, $q_1 = k-3, q_2 = 1$. *The following assertions hold:*

(i) *If* $5 \le k < n-2$, *then*

$$Q_2(\delta,\mathit{\Delta},q_1,q_2) - Q_1(\delta,\mathit{\Delta},q_1,q_2) > 2, Q_4(\delta,\mathit{\Delta},q_1,q_2) - Q_2(\delta,\mathit{\Delta},q_1,q_2) > 2,$$
$$Q_n(\delta,\mathit{\Delta},q_1,q_2) - Q_4(\delta,\mathit{\Delta},q_1,q_2) > 2.$$

(ii) *If* $5 \le k = n-2$ *then*

$$Q_1(\delta,\mathit{\Delta},q_1,q_2) < Q_2(\delta,\mathit{\Delta},q_1,q_2) < Q_4(\delta,\mathit{\Delta},q_1,q_2) < Q_n(\delta,\mathit{\Delta},q_1,q_2) - 1.$$

(iii) *If* $4 = k < n-1$ *then*

$$Q_1(\delta,\mathit{\Delta},q_1,q_2) < Q_2(\delta,\mathit{\Delta},q_1,q_2) < Q_4(\delta,\mathit{\Delta},q_1,q_2) < Q_n(\delta,\mathit{\Delta},q_1,q_2).$$

Proof. From the proof of lemma 2.6 we have

$$Q_2(\delta,\mathit{\Delta},q_1,q_2) - Q_1(\delta,\mathit{\Delta},q_1,q_2) = \binom{n-3}{k-1}$$
$$Q_4(\delta,\mathit{\Delta},q_1,q_2) - Q_2(\delta,\mathit{\Delta},q_1,q_2) = \binom{n-5}{k-4}$$
$$Q_n(\delta,\mathit{\Delta},q_1,q_2) - Q_4(\delta,\mathit{\Delta},q_1,q_2) = \binom{n-5}{k-3} + \binom{n-4}{k-2}$$

The statement of the corollary immediately follows.

Lemma 2.8. *If* $W_{\Omega_A} = E_n^k, 4 < k < n-1,$ *then for* $\delta = (0,0,0,1,1,\dots,1)$, $\Delta = \{2,3,\dots,n-1\}, 1 \le q_1 < k-3, q_2 = 1$ *we have* $Q_1(\delta,\Delta,q_1,q_2) \le Q_2(\delta,\Delta,q_1,q_2) \le Q_4(\delta,\Delta,q_1,q_2) \le Q_n(\delta,\Delta,q_1,q_2)$.

Proof. By definition,

$$B(\Omega,\delta,\Delta,q_1,q_2) = \begin{cases} 1, & \|\delta^1 + \omega_\Omega^1\| \le n-2-q_1, \|\delta^2 + \omega_\Omega^2\| \le 1 \\ 0, & otherwise \end{cases}$$

From $1 \le q_1 < k-3$ it follows that

$$\left|\{\Omega \in \Omega_A \mid i \in \Omega, \|\delta^1 + \omega_\Omega^1\| > n-2-q_1, \|\delta^2 + \omega_\Omega^2\| \le 1\}\right|$$
$$\le \left|\{\Omega \in \Omega_A \mid i \in \Omega, \|\delta^1 + \omega_\Omega^1\| > n-k+2, \|\delta^2 + \omega_\Omega^2\| \le 1\}\right| = 0$$

for each $i = 1,2,\dots,n$.

Hence, we only consider $\left|\{\Omega \in \Omega_A \mid i \in \Omega, \|\delta^2 + \omega_\Omega^2\| > 1\}\right|$ for $i = 1,2,4,n$, and

$$\bar{Q}_1(\delta,\Delta,q_1,q_2) = \binom{n-2}{k-1}, \bar{Q}_2(\delta,\Delta,q_1,q_2) = \binom{n-3}{k-2},$$
$$\bar{Q}_4(\delta,\Delta,q_1,q_2) = \binom{n-3}{k-2}, \bar{Q}_n(\delta,\Delta,q_1,q_2) = 0.$$

Thus,
$Q_1(\delta,\Delta,q_1,q_2) \le Q_2(\delta,\Delta,q_1,q_2) \le Q_4(\delta,\Delta,q_1,q_2) \le Q_n(\delta,\Delta,q_1,q_2)$ and the proof is completed.

The following lemmas can be proven similarly.

Lemma 2.9. *If* $W_{\Omega_A} = E_n^1$ *or* $W_{\Omega_A} = E_n^{n-1}$, *then for* $\delta = (0,0,0,1,1,\dots,1)$, $\Delta = \{2,3,\dots,n-1\}, q_1 = q_2 = 1$ *we have* $Q_1(\delta,\Delta,q_1,q_2) \le Q_2(\delta,\Delta,q_1,q_2) \le Q_4(\delta,\Delta,q_1,q_2) \le Q_n(\delta,\Delta,q_1,q_2)$.

Lemma 2.10. *Let* $\delta = (0,0,0,1,1,\dots,1), \Delta = \{2,3,\dots,n-1\}, q_1 = n-5, q_2 = 1, n > 6$. *If* $W_{\Omega_A} = E_n^1$ *then* $Q_1(\delta,\Delta,q_1,q_2) \le Q_2(\delta,\Delta,q_1,q_2) \le Q_4(\delta,\Delta,q_1,q_2), |Q_n(\delta,\Delta,q_1,q_2) - Q_4(\delta,\Delta,q_1,q_2)| \le 1$. *If* $W_{\Omega_A} = E_n^{n-1}$ *then* $Q_1(\delta,\Delta,q_1,q_2) = Q_2(\delta,\Delta,q_1,q_2) = Q_4(\delta,\Delta,q_1,q_2) = n-2$, $Q_n(\delta,\Delta,q_1,q_2) = n-1$.

Proposition 2.11. *If* Ω_A *is absolutely symmetric then*

$$R(\Omega_A) = \begin{cases} 2, & if\ W_{\Omega_A} = E_n^1\ or\ W_{\Omega_A} = E_n^{n-1} \\ 3, & if\ W_{\Omega_A} = E_n^1 \cup E_n^{n-1} \\ 4, & otherwise \end{cases}$$

Proof. The first case follows from proposition 2.2. Fix (δ,Δ,q_1,q_2) and consider the second case. Note that if $\{Q_i(\delta,\Delta,q_1,q_2)\}_{i=1}^n = \{a,b\}$ for $W_{\Omega_A} = E_n^{n-1}$, then $|a-b| = 1$. Since $|\{Q_i(\delta,\Delta,q_1,q_2)\}_{i=1}^n| = 2$ for $W_{\Omega_A} = E_n^1$ implies that $\{Q_i(\delta,\Delta,q_1,q_2)\}_{i=1}^n = \{0,1\}$, we have $|\{Q_i(\delta,\Delta,q_1,q_2)\}_{i=1}^n| < 4$ for $W_{\Omega_A} = E_n^1 \cup E_n^{n-1}$. Again taking $\delta = (0,0,0,1,1,\dots,1), \Delta = \{2,3,\dots,n-1\}$, $q_1 = q_2 = 1$ yields $R(\Omega_A) = 3$.

For the third case, first suppose that $W_{\Omega_A} \cap (E_n^2 \cup E_n^3 \cup E_n^4) = \emptyset$. Let $\delta = (0,0,0,1,1,\dots,1)$, $\Delta = \{2,3,\dots,n-1\}$, $q_1 = \min\{|\Omega| \mid \Omega \in \Omega_A, |\Omega| \ne 1, |\Omega| \ne n-1\} - 3$, $q_2 = 1$. If $W_{\Omega_A} \cap (E_n^1 \cup E_n^{n-1}) = \emptyset$ then

$$Q_1(\delta,\Delta,q_1,q_2) < Q_2(\delta,\Delta,q_1,q_2) < Q_4(\delta,\Delta,q_1,q_2) < Q_n(\delta,\Delta,q_1,q_2),$$

and therefore $R(\Omega_A) = 4$, follows straight from corollary 2.7 and lemma 2.8. Now if $W_{\Omega_A} \cap (E_n^1 \cup E_n^{n-1}) \neq \emptyset$ then from corollary 2.7, lemma 2.8, and lemma 2.10 follows that we again have

$$Q_1(\delta, \Delta, q_1, q_2) < Q_2(\delta, \Delta, q_1, q_2) < Q_4(\delta, \Delta, q_1, q_2) < Q_n(\delta, \Delta, q_1, q_2)$$

and $R(\Omega_A) = 4$.

Finally, if $W_{\Omega_A} \cap (E_n^2 \cup E_n^3 \cup E_n^4) \neq \emptyset$, we choose $\delta = (0,0,0,1,1,\dots,1)$, $\Delta = \{2,3,\dots,n-1\}, q_1 = q_2 = 1$ and $R(\Omega_A) = 4$ follows at once from corollary 2.5, corollary 2.7, lemma 2.8, and lemma 2.9.

3. A General Method for Constructing Effective Algorithms. Let Ω_A be a supporting set, $M \subseteq N \equiv \{1,2,\dots,n\}$, and G be a subgroup of the symmetric group S_M. For $\sigma \in G$ and $\omega_\Omega = (\omega_1, \omega_2, \dots, \omega_n) \in W_{\Omega_A}$ define $\sigma\omega_\Omega = (\upsilon_1, \upsilon_2, \dots, \upsilon_n)$ as

$$\upsilon_i = \begin{cases} \omega_i, & if\ i \notin M \\ \omega_{\sigma^{-1}(i)}, & if\ i \in M \end{cases}$$

Definition 3.1. *$G(\Omega_A) = \{\sigma \in S_N \mid \sigma\omega_\Omega \in W_{\Omega_A}\}$ is the invariant group of Ω_A.*

For $\sigma \in S_M$ define $\bar{\sigma} \in S_N$ as

$$\bar{\sigma}(i) = \begin{cases} i, & if\ i \notin M \\ \sigma(i), & if\ i \in M \end{cases}$$

For $G \leq S_M$ denote $\bar{G} = \{\bar{\sigma} \mid \sigma \in G\}$.

Definition 3.2. *Let $G \leq S_M$. We say Ω_A is invariant under G over M, if $\bar{G} \leq G(\Omega_A)$.*

Definition 3.3.*We say that Δ-rank of Ω_A over M is equal to k and write $R_\Delta^M(\Omega_A) = k$, if $|\{Q_i(\delta, \Delta, q_1, q_2) \mid i \in M\}| \leq k$, for $\forall\delta \in E^n, \Delta = \{i \mid \delta_i = 1\}, \forall q_1, q_2 \geq 0$, and $|\{Q_i(\delta^0, \Delta^0, q_1^0, q_2^0) \mid i \in M\}| = k$ for some $(\delta^0, \Delta^0, q_1^0, q_2^0)$, $\Delta^0 = \{i \mid \delta_i^0 = 1\}$.*

Theorem 3.4. *Suppose $N_1 \cup N_2 \cup \dots \cup N_k = N, N_i \cap N_j = \emptyset, i \neq j$, $\bar{G}_1 \times \bar{G}_2 \times \dots \times \bar{G}_k \leq G(\Omega_A)$, so that Ω_A is invariant under G_i over N_i, and $R_\Delta^{N_i}(\Omega_A) \leq k_i$. Then $R_\Delta(\Omega_A) \leq \sum_{i=1}^k k_i$.*

The case $G_i = S_{N_i}$ is considered in [2]. Thus, this is a natural generalization of [2]. It is proven that for $G_i = S_{N_i}$ we have $R_\Delta^{N_i}(\Omega_A) \leq 2$. Let us consider another example. Suppose $M = \{i_0, i_1, \dots, i_{m-1}\} \subseteq N, i_0 < i_1 < \dots < i_{m-1}$ and π_M is the cyclic permutation $\pi_M = (i_0 i_1 \dots i_{m-1})$ defined over M. Denote $C_M = <\pi_M> = \{\pi_M^t \mid t \in Z\}$ and

$W_{\Omega_A}^M = \{(\omega_{i_0}, \omega_{i_1}, \dots, \omega_{i_{m-1}}) \mid (\omega_1, \omega_2, \dots, \omega_n) \in W_{\Omega_A}\}$.

Definition 3.5. A system Ω_A of supporting sets is circulant on M if $W_{\Omega_A}^M = \{\sigma\omega^0 \mid \sigma \in C_M\}$, where $\omega^0 = (1,1,\dots,1,0,0,\dots,0) \in E^m, \|\omega^0\| = k, 1 \leq k \leq m-1$. The weight of M is denoted by $\psi(M)$ and is equal to k.

Clearly, if Ω_A is circulant on M then Ω_A is invariant under C_M over M. Let Ω_A be circulant on M and $\psi(M) = k$. For $j \in \{0,1,\dots,m-1\}$ and $\delta = (\delta_1, \delta_2, \dots, \delta_n)$ define

$$l(M,\delta,j) = \left(\delta_{i_{(j-k+1)\, mod\, m}}, \delta_{i_{(j-k+2)\, mod\, m}}, \dots, \delta_{i_{(j+k-1)\, mod\, m}}\right)$$
$$= (l_1, l_2, \dots, l_{2k-1}).$$

For example if $n = 10, M = \{1,2,\dots,10\}, k = 3, l(M,\delta,5) = (\delta_4, \delta_5, \delta_6, \delta_7, \delta_8)$ and $l(M,\delta,9) = (\delta_8, \delta_9, \delta_{10}, \delta_1, \delta_2)$. For $x = (x_1, x_2, \dots, x_{2k-1}) \in E^{2k-1}$ denote $y_i = (x_i, x_{i+1}, \dots, x_{i+k-1}), 1 \le i \le k$ and consider the multiset $W(x) = \{\|y_1\|, \|y_2\|, \dots, \|y_k\|\}$. The following proposition follows from the definition of $B(\Omega, S, S')$:

Proposition 3.6. *If* $W\big(l(M,\delta,j)\big) = W\big(l(M,\delta,k)\big)$ *for some* $j, k \in \{0,1,\dots,m-1\}$, *then for* $\Delta = \{i \mid \delta_i = 1\}$ *and* $q_1, q_2 \ge 0$ *we have* $Q_{i_j}(\delta, \Delta, q_1, q_2) = Q_{i_k}(\delta, \Delta, q_1, q_2)$.

Define an equivalence relation on vectors of length $2k-1$: $x \sim y$ if and only $W(x) = W(y)$. Denote the number of equivalence classes by c_k.

Proposition 3.7. *If Ω_A is circulant on M, then $R_\Delta^M(\Omega_A) \le c_k$.*

Thus, finding the number of equivalence classes gives an upper bound on the number of distinct values of $Q_i(\delta, \Delta, q_1, q_2)$ on M.

Proposition 3.8. $c_k = (k+3)2^{k-2}$.

Proof. Say $H_1, H_2, \dots, H_{C_k}$ are the equivalence classes: $W(x) = W(y)$ for $x, y \in H_j, j = 1,2,\dots,c_k$. Consider sequences $a_1 a_2 \dots a_k$ that satisfy the following conditions:

$$\begin{cases} 0 \le a_i \le k, i = 1,2,\dots,k \\ a_i \le a_{i+1} \le a_i + 1, i = 1,2,\dots,k-1 \end{cases} \tag{3.1}$$

Let us show that for each sequence that satisfies (3.1) there is a class H_j, such that $a_i = \|(x_i, x_{i+1}, \dots, x_{i+k-1})\|, i = 1,2,\dots,k$ for some $x \in H_j$.

We apply induction on k. For $k = 1$ the assertion holds. Now let $k > 1$ and assume the assertion is true for smaller values. There are the following two cases to consider:

Case 1: $a_{k-1} < k$.

By the induction hypothesis there is $x \in E^{2k-3}$ such that $a_i = \|(x_i, x_{i+1}, \dots, x_{i+k-2})\|, i = 1,2,\dots,k-1$. Then for $\tilde{x} = (x_1, x_2, \dots, x_{k-2}, 0, x_{k-1}, \dots, x_{2k-3}, a_k - a_{k-1}) \equiv (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_{2k-1})$ we have $a_i = \|(\tilde{x}_i, \tilde{x}_{i+1}, \dots, \tilde{x}_{i+k-1})\|, i = 1,2,\dots,k$.

Case 2: $a_{k-1} = k$.

Now we have $a_{k-1} = a_k = k$. Consider $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_{2k-1})$ where

$$\tilde{x}_i = \begin{cases} 1 - (a_{i+1} - a_i), & if\ 1 \le i \le k-2 \\ 1, & if\ k-1 \le i \le 2k-1 \end{cases}$$

Note that $a_i = \|(\tilde{x}_i, \tilde{x}_{i+1}, \dots, \tilde{x}_{i+k-1})\|, i = 1,2,\dots,k$.

Thus, there is one to one correspondence between the equivalence classes and the set of sequences satisfying (3.1). Hence, it all comes down to counting the number of sequences satisfying (3.1).

Let S_k denote the set of all sequences that suffice (3.1). We consider two cases similar to what we just have considered. If $a_1, a_2, \dots, a_k \in S_k$, then sequences $a_1, a_2, \dots, a_k, a_k$ and $a_1, a_2, \dots, a_k, a_k + 1$ belong to S_{k+1}. Thus, in

S_{k+1} it remains to count the number of sequences ending with two $k+1$, i.e. the number of sequences satisfying

$$\begin{cases} 0 \leq a_i \leq k+1, i = 1,2,\dots,k+1 \\ a_i \leq a_{i+1} \leq a_i + 1, i = 1,2,\dots,k-1 \\ a_k = a_{k+1} = k+1 \end{cases} \tag{3.2}$$

For each such sequence there is $x = (x_1, x_2, \dots, x_{2k+1}) \in E^{2k+1}$ for which $a_i = \|(x_i, x_{i+1}, \dots, x_{i+k})\|, 1 \leq i \leq k+1$. From $a_k = a_{k+1} = k+1$ it follows that $x_i = 1, i = k, k+1, \dots, 2k+1$. The latter implies that the $a_i \leq a_{i+1} \leq a_i + 1, i = 1,2,\dots,k-1$ condition is met for every value of $x_i, i = 1,2,\dots k-1$. Note that for each $(x_1, x_2, \dots, x_{k-1})$ we get a different sequence. Hence the number of sequences satisfying (3.2) is 2^{k-1}. Combining the two cases we get $c_{k+1} = 2c_k + 2^{k-1}$. Solving the recurrence relation proves the proposition.

Corollary 3.9. *Let $N_1 \cup N_2 \cup \dots \cup N_k = N, N_i \cap N_j = \emptyset, i \neq j$ and Ω_A be circulant on $N_i, n_i = |N_i|$, with $\psi(N_i) = k_i, i = 1,2,\dots,k$. Then $R_\Delta(\Omega_A) \leq \sum_{i=1}^{k} min(n_i, (k_i+3)2^{k_i-2})$.*

The upper bound given in corollary 3.9 is only helpful if $\psi(N_i)$ are very small compared to $|N_i|$. For these cases we may even have equality, i.e. the upper bound in corollary 3.9 is achievable.

Proposition 3.10. The upper bound in corollary 3.9 is exact.

Proof. Let $n = 4t, t > 3, k = 2, N_1 = \left\{1,2,\dots,\frac{n}{2}\right\}$,

$N_2 = \left\{\frac{n}{2}+1, \frac{n}{2}+2, \dots, n\right\}, \psi(N_1) = 1, \psi(N_2) = 2.$

Choose $q_1 = 0, q_2 = 1, \delta = (\delta_1, \delta_2, \dots, \delta_n)$, where

$$\delta_i = \begin{cases} 1, & if\ 1 \leq i \leq \frac{n}{4}\ or\ i = \frac{n}{2}+1\ or\ \frac{n}{2}+3 \leq i \leq \frac{n}{2}+\frac{n}{4}+1 \\ 0, & otherwise \end{cases}$$

For example, if $n = 16, \delta = (1,1,1,1,0,0,0,0,1,0,1,1,1,0,0,0)$.

Then $Q_1(\delta, \Delta, q_1, q_2) = \frac{n}{4}+2, Q_{\frac{n}{4}+1}(\delta, \Delta, q_1, q_2) = \frac{n}{4}-2,$

$Q_{\frac{n}{2}+1}(\delta, \Delta, q_1, q_2) = \frac{n}{2}, Q_{\frac{n}{2}+3}(\delta, \Delta, q_1, q_2) = \frac{n}{2}+\frac{n}{4},$

$Q_{\frac{n}{2}+4}(\delta, \Delta, q_1, q_2) = n, Q_{\frac{n}{2}+\frac{n}{4}+2}(\delta, \Delta, q_1, q_2) = \frac{n}{4}, Q_{\frac{n}{2}+\frac{n}{4}+3}(\delta, \Delta, q_1, q_2) = 0,$

and $R_\Delta(\Omega_A) = (k_1+3)2^{k_1-2} + (k_2+3)2^{k_2-2} = 7.$

Theorem 3.11. *Suppose $N_1 \cup N_2 \cup \dots \cup N_k = N, N_i \cap N_j = \emptyset, i \neq j, \Omega_A$ is invariant under $S_{N_i}, i = 1,2,\dots,t$, and is circulant on $N_j, n_j = |N_j|$, with $\psi(N_j) = k_j, j = t+1, t+2, \dots, k$. Then $R_\Delta(\Omega_A) \leq 2t + \sum_{j=t+1}^{k} \min(n_j, (k_j+3)2^{k_j-2})$.*

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On Effective Implementation of Recognition Algorithms for Calculating Estimates

In this paper the exact values of the ranks for some systems of supporting sets are calculated. A general method for constructing effective algorithms is suggested.

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Ճանաչողական ալգորիթմների արդյունավետ իրականացումը գնահատականների հաշվման համար

Հաշվվում են ռանգերի ճշգրիտ արժեքները որոշ օգնող բազմությունների համար: Առաջարկվում է արդյունավետ ալգորիթմների կառուցման ընդհանուր մեթոդ:

Д. С. Саргсян

Об эффективной реализации алгоритмов распознавания для вычисления оценок

Вычислены конкретные значения рангов для некоторых систем опорных множеств. Предлагается общий подход построения эффективных алгоритмов.

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