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On Dirichlet Type Spaces A^2_ω over the Half-Plane

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1. Introduction. This paper is devoted to an analysis of some Dirichlet type Hilbert spaces A^2_ω of functions holomorphic in the upper half-plane $G^+ := \{z : \operatorname{Im} z > 0\}$, which are similar to the case $p = 2$, $\gamma = 0$ of the considered in [1, 2] larger spaces $A^2_{\omega,\gamma}$, though the new spaces are included in the Hardy space H^2 over G^+. Initially, some extensions of the statements of Theorems 4, 5 and 6 of [2] are proved, which relate with the orthogonal projection and isometry. Then, the spaces $A^2_\omega \subset H^2$ are introduced by the requirement that the derivative of a function belongs to a larger space $A^2_{\omega,0}$, and some results on representations, boundary properties, isometry with H^2, interpolation, biorthogonal systems and bases are obtained for the spaces $A^2_\omega \subset H^2$. The reproducing kernel of the considered spaces A^2_ω, which also provides the approximations, in a sense is better than the Cauchy kernel or rational functions [3], since its singularity on the real axis can be integrable.

2. Extension of the Results on $A^2_{\omega,0}$ Spaces. This section gives some extension of the statements of Theorems 4, 5 and 6 of [2] to some spaces $A^2_{\omega,0}$, where the functions are restricted by some other condition at ∞. We start by several definitions from [1, 2].

Definition 2.1. $A^2_{\omega,\gamma}$ $(0 < p < +\infty, -\infty < \gamma \le 2)$ is the set of those holomorphic in G^+ functions f which satisfy the Nevanlinna type condition

$$\liminf_{R\to+\infty} \frac{1}{R}\int_\beta^{\pi-\beta} \log^+\left|f\left(\mathrm{Re}^{i\theta}\right)\right|\left(\sin\frac{\pi(\vartheta-\beta)}{\pi-2\beta}\right)^{1-\pi/\chi} d\vartheta = 0, \qquad (2.1)$$

For sufficiently small $\rho > 0$ and $\beta = \arcsin\frac{\rho}{R} = \frac{\pi}{2} - \chi$, and simultaneously

$$\|f\|^p_{p,\omega,\gamma} := \iint_{G^+} |f(z)|^p \frac{d\mu_\omega(z)}{(1+|z|)^\gamma} < +\infty, \tag{2.2}$$

where $d\mu_\omega(x+iy) = dx d\omega(2y)$ and it is supposed that ω is of a class Ω_α $(-1 \le \alpha < +\infty)$, i.e. ω is given in $[0,+\infty)$ and satisfies the following conditions:

(i) ω is non-decreasing in $[0,+\infty)$, $\omega(0)=0$ and there exists a strictly decreasing sequence $\delta_k \downarrow 0$ such that also $\omega(\delta_k)$ is strictly decreasing;

(ii) $\omega(t) \succ\prec t^{1+\alpha}$ for some $\Delta_0 \ge 0$ and any $\Delta_0 \le t < +\infty$

($f(t) \succ\prec g(t)$ means that $m_1 f(t) \le g(t) \le m_2 f(t)$ for some constants $m_{1,2} > 0$). The Lebesgue space $L^p_{\omega,\gamma}$ is assumed to be the set of those functions in G^+, which satisfy only the condition (2.2).

Note that $A^2_{\omega,\gamma}$ $(1 \le p < +\infty, -\infty < \gamma < 1, \omega \in \Omega_\alpha, \alpha \ge -1)$ is a Banach space with the norm (2.2) (see Proposition 1.2 in [1]) and it becomes a Hilbert space for $p = 2$. Later we shall deal with the M. M. Djrbashian kernel

$$C_\omega(z) := \int_0^{+\infty} e^{izt} \frac{dt}{I_\omega(t)}, \quad I_\omega(t) := \int_0^{+\infty} e^{-tx} d\omega(x) = x\int_0^{+\infty} e^{-tx}\omega(x)dx$$

(see Section 2 of [2]) which for any $\omega \in \Omega_\alpha$ $(-1 \le \alpha < +\infty)$ is holomorphic in G^+ and becomes the $2+\alpha$-order of the ordinary Cauchy kernel when $\omega(t) = t^{1+\alpha}$ $(\alpha > -1)$. Note that if $\widetilde{\omega}$ is the Volterra square of ω, i.e. $\widetilde{\omega}(0) = 0$ and

$$\widetilde{\omega}(x) = \int_0^x \omega(x-t)d\omega(t), \quad 0 < x < +\infty, \tag{2.3}$$

Then $\widetilde{\omega} \in \Omega_{1+2\alpha}$ and $I^2_\omega(x) = I_{\widetilde{\omega}}(x)$ $(0 < x < +\infty)$ by Lemma 4 of [2].

Theorem 2.1. *If* $\omega \in \Omega_\alpha$ $(-1 \le \alpha < +\infty)$ *and* $\omega(0) = 0$, *then:*

1^0. The orthogonal projection of $L^2_{\omega,0}$ *to* $A^2_{\omega,0}$ *can be written in the form*

$$P_\omega f(z) = \frac{1}{2\pi}\iint_{G^+} f(w) C_\omega(z-\overline{w}) d\mu_\omega(w), \quad z \in G^+. \tag{2.4}$$

2^0. The following representations are true for any function $f \in A^2_{\omega,0}$*:*

$$f(z) = \frac{1}{2\pi}\iint_{G^+} f(w) C_\omega(z-\overline{w}) d\mu_\omega(w)$$
$$= \frac{1}{\pi}\iint_{G^+} \operatorname{Re}\{f(w)\} C_\omega(z-\overline{w}) d\mu_\omega(w), \quad z \in G^+. \tag{2.5}$$

Remark 2.1. In the case of power functions $\omega(t) = t^{1+\alpha}$ $(\alpha > -1)$, formulas (2.5) become the representations found in [8] (see also in [9]). For absolutely continuous measures \$d\omega\$ and spaces defined in a somehow different way, over multidimensional tube domains, the first line of (2.5) was obtained in [10, 11].

The next theorem is the analog of the Paley-Wiener Theorem for $A^2_{\omega,0}$.

Theorem 2.2. *If* $\omega \in \Omega_\alpha$ $(-1<\alpha<+\infty)$ *and* $\omega(0)=0$, *then the space* $A^2_{\omega,0}$ *coincides with the set of functions representable in the form*

$$f(z)=\frac{1}{\sqrt{2\pi}}\int_0^{+\infty} e^{izt}\frac{\Phi(t)}{\sqrt{I_\omega(t)}}dt, \quad z\in G^+, \quad \Phi\in L^2(0,+\infty), \qquad (2.6)$$

where $\|\Phi\|_{L^2(0,+\infty)}=\|f\|_{A^2_{\omega,0}}$, *and*

$$\Phi(t)=\frac{1}{\sqrt{I_\omega(t)}}\int_0^{+\infty} e^{-tv}\widehat{f}_v(t)d\omega(2v), \qquad (2.7)$$

where $\widehat{f}_v$ *is the Fourier transform of the function* f *on the level* iv.

Remark 2.2. For somewhat different spaces over tube domains of $\mathbb{C}^n$ with absolutely continuous measures $\omega(t)dt$, an analog of Theorem 2.2 is proved in [10].

Remark 2.3. Let $S=\bigcup_{\alpha=-1}^{+\infty}\Omega_\alpha$. Then, the union of spaces $\bigcup_{\omega\in S}A^2_{\omega,0}$ coincides with the set of all functions representable in the form

$$f(z)=\int_0^{+\infty} e^{itz}\Psi(t)dt, \quad z\in G^+,$$

where $e^{-\varepsilon t}\Psi(t)\in L^2(0,+\infty)$ for any $\varepsilon>0$.

Theorem 2.3. *If* $\omega\in\Omega_\alpha$ $(-1<\alpha<+\infty)$, $\omega(0)=0$ *and* $\widetilde{\omega}$ *is the Volterra square of* ω *(2.3), then the space* $A^2_{\widetilde{\omega},0}$ *coincides with the set of all functions representable in* G^+ *in the form*

$$f(z)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} C_\omega(z-t)\varphi(t)dt, \quad \text{where} \quad \varphi\in L^2(-\infty,+\infty). \qquad (2.8)$$

For any $f\in A^2_{\widetilde{\omega},0}$, *the function*

$$\varphi_0(z)=L_\omega f(z):=\int_0^{+\infty} f(z+i\sigma)d\omega(\sigma)$$

is the unique one in the Hardy space H^2 *over* G^+, *for which (2.8) is true.* Besides, $\|\varphi_0\|_{H^2}=\|f\|_{A^2_{\widetilde{\omega},0}}$, and $\varphi-\varphi_0$ is orthogonal to H^2 for any function $\varphi\in L^2(-\infty,+\infty)$ which provides the representation (2.8). Further, the operator L_ω is an isometry $A^2_{\widetilde{\omega},0}\to H^2$, and the integral (2.8) defines L^{-1}_ω in H^2.

3. Definition of Dirichlet type spaces A^2_ω**, representations, isometry, a boundary property.** Now we introduce some Dirichlet type spaces A^2_ω which are subsets of the Hardy space H^2 in G^+. Then we prove some representations, an isometry formula and some boundary properties of functions from A^2_ω.

Definition 3.1. Assuming that $\omega_0\in\Omega_\alpha$ $(0\le\alpha<+\infty)$ is continuously differentiable in $(0,+\infty)$ and $\omega_0(x)\ge Mx$ $(0<x<+\infty)$ with some $M>0$, we set

$$\omega_0(x):=\omega_0{}'(x) \text{ and } \omega_1(x):=\int_{+0}^{x}\omega_0(x-t)d\omega_0(t), \quad 0<x<+\infty,$$

and define A^2_ω as the set of those functions f holomorphic in G^+, for which

$$f'\in A^2_{\omega_1,0} \text{ and } \lim_{y\to+\infty} f(x+iy)=0, \quad -\infty<x<+\infty. \qquad (3.1)$$

Also, we set $\|f\|_{A^2_\omega} := \|f'\|_{A^2_{\omega_1,0}}$.

Note that the above definition is correct, since $\omega_1 \in \Omega_{1+2\alpha}$ by Lemma 4 of [2]. *Everywhere below, the functions ω, ω_0 and ω_1 are assumed to be as above.* Further, one can see that Lemma 4.2 in [12] is true also when there $\alpha = 0$, and therefore, if $\omega = \omega_0'$ is a positive, nonincreasing in $(0,+\infty)$ function such that $\int_0^1 \{t\omega_0'(t)\}^{-1} dt < +\infty$, then for any $z = x + iy \in G^+$

$$C_\omega(z) = L_\gamma\left(\frac{1}{-iz}\right) = \int_0^{+\infty} \frac{d\gamma(t)}{-i(z+it)},$$

where γ is a nondecreasing function in $(0,+\infty)$, such that $\gamma(0) = 0$ and $\gamma(t) \le [\omega_0'(t)]^{-1}$, $0 < t < +\infty$, and hence the function C_ω is holomorphic out of the negative imaginary half-axis, and the origin is an integrable singularity.

One of the representations of the next theorem is an analog of that of the Paley-Wiener Theorem (see, eg. [13], Theorem 11.9 at p. 186), while the other one gives an explicit isometry between A^2_ω and the Hardy space H^2 over the half-plane.

Theorem 3.1. 1^0. *A^2_ω is a Hilbert space and $A^2_\omega \subset H^2$. Besides, A^2_ω coincides with the set of all functions representable in the form*

$$f(z) = \frac{1}{\sqrt{2\pi i}} \int_0^{+\infty} e^{izt} \frac{\Phi(t)}{\sqrt{I_\omega(t)}} dt, \quad z \in G^+, \tag{3.2}$$

where $\Phi \in L^2(0,+\infty)$, and $\quad \|\Phi\|_{L^2(0,+\infty)} = \|f\|_{A^2_\omega}$.

2^0. *A^2_ω coincides with the set of all functions representable in G^+ as*

$$f(z) = \frac{1}{2\pi i} \int_{-\infty}^{+\infty} C_\omega(z-t)\varphi(t)dt, \quad \varphi(z) \in H^2, \tag{3.3}$$

where $\|\varphi\|_{H^2} = \|f\|_{A^2_\omega}$. Formula (3.3) defines an isometry $H^2 \to A^2_\omega$, the inversion of which is

$$\varphi(z) = L_\omega f(z) := \int_0^{+\infty} f'(z+it)\omega(t)dt, \quad z \in G^+. \tag{3.4}$$

The functions of the Dirichlet type spaces A^2_ω possess nontangential boundary values out of some exceptional, zero omega-capacity sets on the real axis. Note that the mentioned omega-capacity is introduced in [14] as a generalization of the considered in [7] half-plane analog of Frostman's well-known alpha-capacity. Below, we give a somehow modified, but equivalent to that of [14] definition.

Definition 3.2. Let $E \subseteq (-\infty,+\infty)$ be a Borel measurable set. Then E is of positive ω-capacity, or $C_\omega(E) > 0$, if for any $R > 0$ there exists a finite Borel measure $\sigma \ge 0$ supported on $E \cap (-R,R)$, $(\sigma \prec E \cap (-R,R))$, such that

$$S_R := \sup_{z\in G^+} \int_{-R}^{R} |C_\omega(z-t)| d\sigma(t) < +\infty.$$

If there is not such a measure, i.e. $S_R = +\infty$ for some $R > 0$ and any finite, nonnegative Borel measure $\sigma \prec E \cap (-R, R)$, then E is of zero ω-capacity, or $C_\omega(E) = 0$.

Proposition 3.1. *Since the functions $f \in A^2_\omega$ possess representations of the form (3.3), Lemma 4.4 of [14] implies that these functions have nontangential boundary values $f(x)$ at all points $-\infty < x < +\infty$, except a set of zero omega-capacity.*

4. Biorthogonal systems, bases and interpolation. The explicit form (3.3) of an isometry between the Hardy space H^2 over the half-plane G^+ and the spaces A^2_ω permits to convert any result of additive character in H^2 into a similar statement in A^2_ω. In particular, for $p = 2$ the results of [15, 16] on biorthogonal systems and interpolation in H^p $(1 < p < +\infty)$ imply some similar statements in A^2_ω. Almost all of these statements are given in the below propositions. For simplicity, we assume that $\{z_k\}_1^\infty$ is a sequence of *pairwise different* points in G^+. It is said that $\{z_k\}_1^\infty \in \Delta$, if the sequence $\{z_k\}_1^\infty$ is uniformly separated, i.e.

$$\inf_{k\geq 1} \prod_{j=1,\ j\neq k}^{\infty} \left| \frac{z_j - z_k}{z_j - \overline{z_k}} \right| = \delta > 0. \tag{4.1}$$

Note that this relation implies the validity of the Blaschke condition

$$\sum_{k=1}^{\infty} \frac{\operatorname{Im} z_k}{1 + |z_k|^2} < +\infty \tag{4.2}$$

which is necessary and sufficient for the convergence of the Blaschke product

$$B(z) = \prod_{k=1}^{\infty} \frac{z - z_k}{z - \overline{z_k}} \frac{|1 + z_k^2|}{1 + z_k^2}$$

with zeros at $\{z_k\}_1^\infty$ to a holomorphic function everywhere in the finite complex plane, except the closure of the set $\{\overline{z_k}\}_1^\infty$.

The inequality (3.21) of [15] is transferred to the following proposition.

Proposition 4.1. *If $\{z_k\}_1^\infty \in \Delta$, then for any function $f \in A^2_\omega$ the following inequality is true: $\sum_{k=1}^{\infty} \operatorname{Im} z_k \left| L_\omega f(z_k) \right|^2 \leq C \|f\|^2$, where $C > 0$ is a constant independent of f.*

Before giving some other propositions on approximation and interpolation in A^2_ω, note that the functions

$$r_k(z) = \frac{1}{z - \overline{z_k}} \quad \text{and} \quad \Omega_k(z) = \frac{B(z)}{z - z_k}, \quad k = 1, 2, ...,$$

are of H^2 in G^+. Hence, all functions $L^{-1}_\omega r_k(z) := r_{k,\omega}(z)$ and $L^{-1}_\omega \Omega_k(z) := \Omega_{k,\omega}(z)$ $(k = 1, 2, ...)$ are of A^2_ω, and one can verify that $r_k(z) = -iC_\omega(z - \overline{z_k})$. Theorem D and some other results of [15] imply the following statement.

Proposition 4.2. *If the sequence* $\{z_k\}_1^\infty$ *does not satisfy the Blaschke condition, i.e. the series (4.2) is divergent, then the systems* $\{-iC_\omega(z-\overline{z_k})\}_1^\infty$ *and* $\{\Omega_{k,\omega}(z)\}_1^\infty$ *are complete in* A_ω^2.

Further, a transformation in the conditions (1.16), (1.17) of [15] (or (2.2), (2.3) of [16]) leads to the introduction of a subset $A_\omega^2\{z_k\}\subset A_\omega^2$ of functions f for which there exist some $g\in H^2$ such that for almost all $-\infty<x<+\infty$ the non-tangential boundary values of $g(-z)B(z)$ from inside the lower half-plane $G^-=\{z:\operatorname{Im}z<0\}$ coincide with those of the function $L_\omega f\in H^2$ form inside G^+. Evidently, $A_\omega^2\{z_k\}\subset A_\omega^2$ can be considered only under the condition (4.2). By Theorem 2 of [16] we get the following proposition.

Proposition 4.3. *The systems* $\{-iC_\omega(z-\overline{z_k})\}_1^\infty$ *and* $\{\Omega_{k,\omega}(z)\}_1^\infty$ *are biorthogonal:*

$$\left(-iC_\omega(z-\overline{z_k}),\,\Omega_{\nu,\omega}(z)\right)_{A_\omega^2}=\begin{cases}1, & if \quad \nu=k,\\ 0, & if \quad \nu\neq k.\end{cases}$$

The next proposition is implied by Lemmas B and 1.1 of [15].

Proposition 4.4. *If* $f\in A_\omega^2$, *then:*

1^0. f *belongs to* $A_\omega^2\{z_k\}$ *if and only if*

$$\Psi(z)=\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{L_\omega f(t)}{B(t)}\frac{dt}{t-z}\equiv 0,\quad z\in G^+,$$

where $L_\omega f(t)$ *and* $B(t)$ *are the boundary values of* $L_\omega f(z)\in H^2$ *and* $B\in H^2$.

2^0. *The following orthogonal decomposition is true:* $f(z)=F(z)+R(z)$ $(z\in G^+)$, $\|f\|_{2,\omega}^2=\|F\|_{2,\omega}^2+\|R\|_{2,\omega}^2$, where $F\in A_\omega^2\{z_k\}$, $R=L_\omega^{-1}[B\Psi]\in A_\omega^2$.

By Theorems 4.1 and 5.2 of [15] we get the next proposition.

Proposition 4.5. *Each of the systems* $\{-iC_\omega(z-\overline{z_k})\}_1^\infty$ *and* $\{\Omega_{k,\omega}(z)\}_1^\infty$ *is a basis in* $A_\omega^2\{z_k\}$ *if and only if* $\{z_k\}_1^\infty\in\Delta$.

Using formulas (4.29), (4.31) and a formula from the proof of Theorem 5.2 in [15] we get the next proposition.

Proposition 4.6. *If* $\{z_k\}_1^\infty\in\Delta$, *then any function* $f\in A_\omega^2\{z_k\}$ *is representable in* G^+ *by both series*

$$f(z)=\sum_{k=1}^\infty c_k(f)C_\omega(z-\overline{z_k})=\sum_{k=1}^\infty L_\omega f(z_k)\Omega_{k,\omega}(z)\quad with\quad c_k(f)=\left(f,\Omega_{k,\omega}\right),$$

which converge in the norm of A_ω^2 *and uniformly inside* G^+.

Theorem 4.2 of [15] implies the next proposition.

Proposition 4.7. *If* $\{z_k\}_1^\infty\in\Delta$, *then any function* $f\in A_\omega^2$ *is representable as* $f(z)=\sum_{k=1}^\infty c_k(f)C_\omega(z-\overline{z_k})+\psi(z)$, *where the series is convergent in* A_ω^2 *and uniformly in* G^+, *and the following inclusions are true:* $\psi(z)=L_\omega^{-1}[B(z)\Psi(z)]$ *and* $\psi(z)=\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{L_\omega f(t)}{B(t)}\frac{dt}{t-z}\in H^2$.

By Theorems 5.1 and 5.2 of [15] we get the next proposition.

Proposition 4.8. *The following statements are true.*

1^0. *If* $\{z_k\}_1^\infty \in \Delta$ *and* $\{w_k\}_1^\infty$ *is a sequence of complex numbers for which* $A = \sum_{k=1}^{\infty} \operatorname{Im} z_k |w_k|^2 < +\infty$, *then there is a unique function* $f_0 \in A_\omega^2\{z_k\}$ *such that* $L_\omega f_0(z_k) = w_k \quad (k = 1, 2, ...)$ *and* $\|f_0\|_{f_0 \in A_\omega^2} \le C_\delta A$, *where* $C_\delta > 0$ *is a constant depending solely on* δ *of (4.1). This function is expanded in the series*

$$f_0(z) = \sum_{k=1}^{\infty} w_k \Omega_{k,\omega}(z), \quad z \in G^+,$$

which converges in the norm of A_ω^2 *and uniformly inside* $z \in G^+$.

2^0. *Conversely, if the set of the sequences* $\left\{(\operatorname{Im} z_k)^{1/2} f'(z_k)\right\}_1^\infty$ *with all possible functions* $f \in A_\omega^2$ *coincides with the space* l^2 *of sequences of complex numbers, which possess finite sums of squares of modules, then* $\{z_k\}_1^\infty \in \Delta$.

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On Dirichlet Type Spaces A_ω^2 over the Half-Plane

Some extensions of the results of the first author related with the Hilbert spaces $\mathrm{A}_{\omega,0}^2$ of functions holomorphic in the half-plane are proved. Some new Hilbert spaces A_ω^2 of Dirichlet type are introduced, which are included in the Hardy space H^2 over the half-plane. Several results on representations, boundary properties, isometry, interpolation, biorthogonal systems and bases are obtained for the spaces $\mathrm{A}_\omega^2 \subset \mathrm{H}^2$.

Ա. Մ. Ջրբաշյան, Ջ. Պեքենդինո

Դիրիխլեի տիպի A_ω^2 տարածություններ կիսահարթությունում

Տրված են համահեղինակներից առաջինի՝ կիսահարթությունում հոլոմորֆ ֆունկցիաների $A_{\omega,0}^2$ տարածություններին վերաբերող որոշ արդյունքների ընդլայնումներ: Ներմուծված են նոր, Դիրիխլեի տիպի, հիլբերտյան A_ω^2 տարածություններ, որոնք պարունակվում են կիսահարթության Հարդիի H^2 տարածության մեջ: $A_\omega^2 \subset H^2$ տարածություններում ստացված են արդյունք-

ներ ներկայացումների, եզրային հատկությունների, իզոմետրիայի, ինտերպո-լյացիայի, բիորթոգոնալ համակարգերի և բազիսների վերաբերյալ:

А. М. Джрбашян, Дж. Пехендино

Пространства A^2_ω типа Дирихле в полуплоскости

Даны расширения некоторых результатов первого из соавторов, относящиеся к пространствам $A^2_{\omega,0}$ функций, голоморфных в полуплоскости. Введены новые гильбертовы пространства A^2_ω типа Дирихле, содержащиеся в пространстве H^2 Харди. В пространствах $A^2_\omega \subset H^2$ получены результаты о представлениях, граничных свойствах, изометрии, интерполяции, биортогональных системах и базисах.

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