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On a Dirichlet Problem for One Properly Elliptic Equation in the Unit Disk

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1. Introduction. Formulation of the results. Let $D=\{(x,y):x^2+y^2<1\}$ be the unit disc of the complex plane. We consider in the domain D fourth order elliptic differential equation

$$\sum_{k=0}^{4} A_k \frac{\partial^4 u}{\partial x^k \partial y^{4-k}}(x,y)=0, \quad (x,y)\in D, \tag{1}$$

where A_k are the complex constants ($A_0\neq 0$). We suppose that the roots λ_j $(j=1,2,3,4)$ of the characteristic equation

$$\sum_{k=0}^{4} A_k \lambda^{4-k}=0, \tag{2}$$

satisfy the condition

$$\lambda_1=\lambda_2\neq i,\ \Im\lambda_1>0;\ \lambda_3\neq\lambda_4,\ \lambda_j\neq -i, \Im\lambda_j<0,\ j=3,4, \tag{3}$$

that is the equation (1) is properly elliptic. We want to find the solution of the equation (1) in the class $C^4(D)\bigcap C^{(1,\alpha)}(\overline{D})$, which satisfy the Dirichlet conditions

$$u\big|_\Gamma = f(x,y), \quad \frac{\partial u}{\partial N}\bigg|_\Gamma = g(x,y), \quad (x,y)\in\Gamma. \tag{4}$$

on the boundary Γ. Here $f\in C^{(1,\alpha)}(\Gamma)$ and $g\in C^{(\alpha)}(\Gamma)$ are the given functions, $\frac{\partial}{\partial N}=-\frac{\partial}{\partial r}$ - is a differentiation in the inner normal direction to the boundary Γ (here and further $z=x+iy=re^{i\varphi}$).

It is well known ([1, 2]) that for the properly elliptic equation (1) the Dirichlet problem is Fredholmian. We want to determine the defect numbers of the problem (1), (4), that is the number of the linearly independent solutions of the homogeneous problem (the problem (1), (4) where $f\equiv g\equiv 0$) and the number of the linearly independent solvability conditions of the inhomogeneous problem. The problem (1), (4) in the case when one of the roots of the characteristic equation (2) is equal to $\pm i$ was completely investigated in [23]. The case $\lambda_1 = \lambda_2, \lambda_3 = \lambda_4$ was considered in [4]. In [5] was investigated the problem (1), (4) for higher order equation (1).

In this paper we consider the problem (1), (4) (homogeneous and inhomogeneous), when the roots of the equation (2) satisfy the conditions (3). We get the new formula for the determination of the defect numbers of the problem and we find in explicit form the solutions of homogeneous problem and the solvability conditions of inhomogeneous problem.

For exact formulation of the obtained results let's represent the equation (1) and boundary conditions (3) in the complex form, using operators of complex differentiation

$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right), \ \frac{\partial}{\partial \overline{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right).$$

Taking into account the conditions (3) the equation (1) will be represented in following form:

$$\left(\frac{\partial}{\partial \overline{z}} - \mu\frac{\partial}{\partial z}\right)^2 \left(\frac{\partial}{\partial z} - \nu_1\frac{\partial}{\partial \overline{z}}\right)\left(\frac{\partial}{\partial z} - \nu_2\frac{\partial}{\partial \overline{z}}\right)u(x,y) = 0, \tag{3}$$

where $\mu = \dfrac{i-\lambda_1}{i+\lambda_1}$, $\nu_j = \dfrac{i+\lambda_{2+j}}{i-\lambda_{2+j}}$ $j = 1,2$. Using the conditions (3), we have

$$|\mu| < 1, \nu_1 \neq \nu_2, \ |\nu_j| < 1, \ \ j = 1,2; \ \ \mu\nu_1\nu_2 \neq 0. \tag{4}$$

Boundary conditions (4) are reduced to equivalent form ([3])

$$\left.\frac{\partial u}{\partial \overline{z}}\right|_{\Gamma} = F(x,y), \ \left.\frac{\partial u}{\partial z}\right|_{\Gamma} = G(x,y), \ (x,y) \in \Gamma; \ u(1,0) = f(1,0). \tag{5}$$

Here the functions F and G from the class $C^{(\alpha)}(\Gamma)$ are determined by the formulas

$$F(x,y) = \frac{z}{2}\left(g(x,y) + i\frac{\partial f}{\partial \varphi}(x,y)\right), \ G(x,y) = \frac{\overline{z}}{2}\left(g(x,y) - i\frac{\partial f}{\partial \varphi}(x,y)\right), \ z = re^{i\varphi} \in \Gamma. \tag{6}$$

Using these denotations the results of the paper may be formulated in following way.

Theorem 1. *Let's denote* $\sigma = \mu\nu_1, \tau = \mu\nu_2$. *The problem (1), (4) is uniquely solvable if and only if the conditions*

$$P_k(\sigma,\tau) \equiv \sum_{m=0}^{k-1}\sum_{p=0}^{m}(m-p)(\sigma\tau)^p \sum_{j=0}^{m-p-1}\sigma^j\tau^{m-p-j} \neq 0, \quad k = 3,4,\ldots \tag{7}$$

hold. If the conditions (7) fail, that is $P_{k_0}(\sigma,\tau) = 0$ *for some value* $k_0 > 2$, *then the homogeneous problem (1), (4) has one linearly independent solution which is polynomial of order* $k_0 + 1$. *The corresponding inhomogeneous problem has a solution if the boundary functions* F, G *satisfy one linearly independent* *orthogonality condition. Therefore, the defect numbers of the problem are equal to the quantity of the numbers* k_0 *for which* $P_{k_0}(\sigma,\tau)=0$.

2. Proof of the theorem 1. The general solution of the equation (3) may be represented in the form ([5]):

$$u(x,y)=\Phi_0(z+\mu\bar{z})+\frac{\partial}{\partial\varphi}\Phi_1(z+\mu\bar{z})+\Psi_0(\bar{z}+\nu_1 z)+\Psi_1(\bar{z}+\nu_2 z), \qquad (8)$$

where Φ_j and Ψ_j $(j=0,1)$ are the functions, which must be determined, analytic in the domains $G=\{z+\mu\bar{z}\mid z\in D\}$ and $D_j=\{\bar{z}+\nu_{j+1}z\mid z\in D\}$ respectively. We substitute the function (8) in the boundary equations (5). Using operator identity ([5])

$$\frac{\partial^{k+m}}{\partial z^k\partial\bar{z}^m}\frac{\partial^l}{\partial\varphi^l}=\left(\frac{\partial}{\partial\varphi}+(k-m)iI\right)^l\frac{\partial^{k+m}}{\partial z^k\partial\bar{z}^m},$$

we get

$$\mu\Phi_0'(z+\mu\bar{z})+\mu\left(\frac{\partial}{\partial\varphi}-iI\right)\Phi_1'(z+\mu\bar{z})+\Psi_0'(\bar{z}+\nu_1 z)+\Psi_1'(\bar{z}+\nu_2 z)=F(z),\ z\in\Gamma,$$

$$\Phi_0'(z+\mu\bar{z})+\left(\frac{\partial}{\partial\varphi}+iI\right)\Phi_1'(z+\mu\bar{z})+\nu_1\Psi_0'(\bar{z}+\nu_1 z)+$$

$$+\nu_2\Psi_1'(\bar{z}+\nu_2 z)=G(z),\ z\in\Gamma. \qquad (9)$$

Now, we represent the function $\Omega(z+\mu\bar{z})$, where Ω is analytic in the domain $G=\{z+\mu\bar{z}:z\in D\}$, on the circumference Γ using analytic in D functions. It was proved in [76] that the function $\Omega(z+\mu\bar{z})$ may be represented in the form (for $|z|=1$)

$$\Omega(z+\mu\bar{z})=\omega(z)+\omega(\mu\bar{z}), \qquad (10)$$

where ω is analytic function in the unit disk. If we have the function ω, then the function Ω is determined by the formula:

$$\Omega(\zeta)=\omega\left(\frac{\zeta+\sqrt{\zeta^2-4\mu}}{2}\right)+\omega\left(\frac{\zeta-\sqrt{\zeta^2-4\mu}}{2}\right), \qquad (11)$$

for $\zeta\in G$. In these formulas we choose the branch of $\sqrt{\zeta^2-4\mu}$, what is analytic outside the segment $\left[-2\sqrt{\mu},2\sqrt{\mu}\right]$ and satisfies the condition $\zeta^{-1}\sqrt{\zeta^2-4\mu}\to 1$ for $\zeta\to\infty$. We use (10) and represent the functions Φ_j',Ψ_j' on the circumference Γ

$$\Phi_j'(z+\mu\bar{z})=\vartheta_j(z)+\vartheta_j(\mu\bar{z})\equiv\sum_{k=0}^{\infty}A_{kj}z^k+\sum_{k=0}^{\infty}A_{kj}\mu^k z^{-k},$$

$$\Psi_j'(\bar{z}+\nu_{j+1}z)=\rho_j(\bar{z})+\rho_j(\nu_{j+1}z)\equiv\sum_{k=0}^{\infty}B_{kj}z^{-k}+\sum_{k=0}^{\infty}B_{kj}\nu_{j+1}^k z^k,\ \ j=0,1,\ \ z\in\Gamma. \qquad (12)$$

We want to determine unknown functions ϑ_j and ρ_j. These functions are analytic in the unit disc D, therefore they determined by the corresponding Taylor coefficients A_{kj} и B_{kj}. For the determination of these coefficients let's substitute the expansions (12) and Fourier expansions of the functions F and G

$$F(z)=\sum_{k=0}^{\infty}F_k z^k+\sum_{k=1}^{\infty}F_{-k}\overline{z}^k \equiv F_+(z)+F_-(z),$$

$$G(z)=\sum_{k=0}^{\infty}G_k z^k+\sum_{k=1}^{\infty}G_{-k}\overline{z}^k \equiv G_+(z)+G_-(z), \qquad (15)$$

in the boundary equations (11). We get

$$\sum_{k=0}^{\infty}A_{k0}\mu z^k+\sum_{k=0}^{\infty}A_{k0}\mu^{k+1}z^{-k}+\sum_{k=0}^{\infty}A_{k1}(ik-i)\mu z^k-\sum_{k=0}^{\infty}A_{k1}(ik+i)\mu^{k+1}z^{-k}+\sum_{k=0}^{\infty}B_{k0}z^{-k}+$$

$$+\sum_{k=0}^{\infty}B_{k0}\nu_1^k z^k+\sum_{k=0}^{\infty}B_{k1}z^{-k}+\sum_{k=0}^{\infty}B_{k1}\nu_2^k z^k=\sum_{k=0}^{\infty}F_k z^k+\sum_{k=1}^{\infty}F_{-k}z^{-k}, \quad |z|=1,$$

$$\sum_{k=0}^{\infty}A_{k0}z^k+\sum_{k=0}^{\infty}A_{k0}\mu^k z^{-k}+\sum_{k=0}^{\infty}A_{k1}(ik+i)z^k+\sum_{k=0}^{\infty}A_{k1}(i-ik)\mu^k z^{-k}+\sum_{k=0}^{\infty}B_{k0}\nu_1 z^{-k}+$$

$$+\sum_{k=0}^{\infty}B_{k0}\nu_1^{k+1}z^k+\sum_{k=0}^{\infty}B_{k1}\nu_2 z^{-k}+\sum_{k=0}^{\infty}B_{k1}\nu_2^{k+1}z^k=\sum_{k=0}^{\infty}G_k z^k+\sum_{k=1}^{\infty}G_{-k}z^{-k}, \quad |z|=1. \qquad (16)$$

Equating the coefficients by the same degrees of z and $\overline{z}$, we get the system for the determination of the unknown coefficients A_{kj} and B_{kj}. For $k=0$ we have:

$$2\mu A_{00}-2\mu i A_{01}+2B_{00}+2B_{01}=F_0,\ 2A_{00}+2iA_{01}+2\nu_1 B_{00}+2\nu_2 B_{01}=G_0. \qquad (13)$$

If $k\ge 1$ we have fourth order system of linear equations for the determination A_{kj} and B_{kj}:

$$A_{k0}+i(k+1)A_{k1}+\nu_1^{k+1}B_{k0}+\nu_2^{k+1}B_{k2}=G_k,$$
$$\mu A_{k0}+i(k-1)\mu A_{k1}+\nu_1^k B_{k0}+\nu_2^k B_{k2}=F_k,$$
$$\mu_1^k A_{k0}-i(k-1)\mu_1^k A_{k1}+\nu_1 B_{k0}+\nu_2 B_{k2}=G_{-k},$$
$$\mu_1^{k+1}A_{k0}-i(k+1)\mu_1^{k+1}A_{k1}+B_{k0}+B_{k2}=F_{-k}, \qquad (14)$$

We consider the determinant of the main matrix of the system (14):

$$S_k=\det\tilde{S}_k\equiv\det\begin{pmatrix}1 & i(k+1) & \nu_1^{k+1} & \nu_2^{k+1}\\ \mu & i(k-1)\mu & \nu_1^k & \nu_2^k\\ \mu^k & i(-k+1)\mu^k & \nu_1 & \nu_2\\ \mu^{k+1} & i(-k-1)\mu^{k+1} & 1 & 1\end{pmatrix}. \qquad (15)$$

After transformation, using denotations of the theorem **1**, we get:

$$S_k=i\begin{vmatrix}1 & k+1 & \sigma^{k+1} & \tau^{k+1}\\ 1 & (k-1) & \sigma^k & \tau^k\\ 1 & (-k+1) & \sigma & \tau\\ 1 & (-k-1) & 1 & 1\end{vmatrix}=i(1-\sigma)^2(1-\tau)^2\sum_{m=0}^{k-1}\sum_{p=0}^{m}(m-p)\left(\sigma^p\tau^m-\sigma^m\tau^p\right)\equiv$$

$$\equiv i(1-\sigma)^2(1-\tau)^2\,\Theta_k(\sigma,\tau). \qquad (16)$$

And finally, the function Θ_k may be represented in the form

$$\Theta_k(\sigma,\tau)=(\tau-\sigma)P_k(\sigma,\tau),$$

where P_k is a function defined in the theorem 1. From the conditions of the theorem we have $\sigma \neq 1$, $\tau \neq 1$ and $\sigma \neq \tau$. Therefore, $S_k \neq 0$ for $k > 2$ if and only if the conditions (9) hold. $S_2 \neq 0$ also, because S_2 is a generalized Vandermonde determinant with different terms.

Let's suppose, that the conditions (9) hold. Then the coefficients A_{kj} and B_{kj} for $k \geq 2$ are uniquely determined. Determinant S_1 of the system (18) for $k = 1$ is equal to zero, because second and third rows are the same. But, taking into account the formulas (8) we have

$$F_1 = G_{-1} = \frac{1}{4\pi}\int_0^{2\pi} g(\cos\varphi, \sin\varphi)d\varphi,$$

that is the system (18) has a solution (not unique) for $k = 1$ also. And at last we determine (not uniquely too) the coefficients A_{0j} and B_{0j} from the system (17). Thus, the coefficients A_{kj} and B_{kj} may be found for arbitrary k, and therefore, we get the functions ϑ_j and ρ_j after what, by the formula (13) we determine Φ_j and Ψ_j. We have $S_k \to -2i$ for $k \to \infty$, hence the coefficients A_{kj} and B_{kj} have the same rate of decreasing as coefficients F_k and G_k, therefore, the resulting function u (the solution of the problem (1), (4), defined by the formula (10)) belongs to the prescribe class $C^{(1,\alpha)}(\Gamma)$. Now, let's consider the homogeneous problem (1), (4). In this case, if the conditions (9) hold, the corresponding Taylor coefficients A_{kj} and B_{kj} are equal to zero for $k > 1$ and, therefore, nontrivial solution of the homogeneous problem (1), (4) may be at most second order polynomial. But the theorem 5.1 from [7] (p. 84) implies, that every nontrivial polynomial, which satisfies homogeneous conditions (4) must be divisible by $(1 - z\overline{z})^2$, that is must have at least fourth order. It means, that if the conditions (9) hold, then the corresponding homogeneous problem has only zero solution.

If the condition (9) fail for some $k_0 > 2$, that is $S_{k_0} = 0$, then the corresponding homogeneous problem has one linearly independent solution which is determined by nonzero solution $A_{k_0 j}$ and $B_{k_0 j}$ of the system (18). Theorem 1 is proved.

3. Some numerical results. In this point we will speak about the conditions (9). First, open the brackets in (5). We get

$$\left(P_3(\sigma,\tau)\frac{\partial^4}{\partial z^2 \partial \overline{z}^2} + E\frac{\partial^4}{\partial z \partial \overline{z}^3} + H\frac{\partial^4}{\partial z^3 \partial \overline{z}} + \mu^2 \frac{\partial^4}{\partial z^4} + \nu_1\nu_2 \frac{\partial^4}{\partial \overline{z}^4} \right) u = 0, \qquad (21)$$

where $P_3(\sigma,\tau)$ defined in (9) for $k = 3$:

$$P_3(\sigma,\tau) = 1 + 2(\sigma + \tau) + \sigma\tau \equiv 1 + 2\mu(\nu_1 + \nu_2) + \mu^2\nu_1\nu_2 ,$$

and the constants E, H are following:

$$E = -(\nu_1 + \nu_2 + 2\mu\nu_1\nu_2); \quad H = -(2\mu + \mu^2(\nu_1 + \nu_2)).$$

Let's illustrate the results of the theorem 1 in the cases $k = 3,4$. We consider the homogeneous problem. Taking into account homogeneous conditions (4) we know ([7], p.84) that the seeking solution must be divisible by $(1-z\bar{z})^2$.

First, let's suppose, that the function $u_0(z,\bar{z})=\alpha(1-z\bar{z})^2$ $(\alpha\neq 0)$ is a solution of the homogeneous problem (1), (4). This function satisfies the homogeneous boundary conditions (4). Substituting the function in the equation (21), we get:

$$P_3(\sigma,\tau)4\alpha=0.$$

Thus, the function u_0 is non-zero solution of the homogeneous problem (1), (4) if and only if $P_3(\sigma,\tau)=0$. For example, if the constants μ, ν_1, ν_2 satisfy the conditions

$$\mu\nu_1=\sigma=-\frac{2}{3},\ \mu\nu_2=\tau=-\frac{1}{4},$$

then the function u_0 is a non-zero solution of the homogeneous problem (1), (4).

Now, let's seek the non-zero solution of the homogeneous problem (1), (4) in the form

$$u_1(z,\bar{z})=(1-z\bar{z})^2(\beta z+\gamma\bar{z}),\qquad |\beta|+|\gamma|\neq 0.$$

Substituting this function in the equation (21), we get:

$$P_3(\sigma,\tau)(12\beta z+12\gamma\bar{z})+12E\gamma z+12H\beta\bar{z}=0.$$

This equality holds if and only if β,γ satisfy the system:

$$P_3(\sigma,\tau)\beta+E\gamma=0,\ H\beta+P_3(\sigma,\tau)\gamma=0. \tag{22}$$

This homogeneous system of linear equations has non-zero solution if and only if the determinant of the main matrix of this system is equal to zero. Using denotations of the theorem 1, $\sigma=\mu\nu_1,\ \tau=\mu\nu_2$, this determinant may be written as follows:

$$P_3^2(\sigma,\tau)-EH=(1+2(\sigma+\tau)+\sigma\tau)^2-(\sigma+\tau+2)(\sigma+\tau+2\sigma\tau)=P_4(\sigma,\tau).$$

Thus, we get that u_1 is the non-zero solution of the homogeneous problem (1), (4) if and only if $P_4(\sigma,\tau)=0$. Let's show that P_4 may be equal zero for σ,τ such that $|\sigma|<1,\ |\tau|<1$. First, we represent $P_4(\sigma,\tau)=1+2(\sigma+\tau)+3\sigma^2+4\sigma\tau+$ $+3\tau^2+2\sigma\tau(\sigma+\tau)+\sigma^2\tau^2$ in the form:

$$P_4(\sigma,\tau)=1+2\sigma+3\sigma^2+2\tau(1+2\sigma+\sigma^2)+\tau^2(3+2\sigma+\sigma^2)\equiv E_0+E_1\tau+E_2\tau^2, \tag{23}$$

fix arbitrary σ ($|\sigma|<1$) and consider second order polynomial

$$p(\tau)=E_0+E_1\tau+E_2\tau^2.$$

We will use Shur transform (see [8], p.492) for showing that two roots of this polynomial lie in the unit disc. We have $p^*(\tau)=\tau^2\overline{p\left(\frac{1}{\bar{\tau}}\right)}=\bar{E}_2+\bar{E}_1\tau+\bar{E}_0\tau^2$ and, therefore, Shur transform Tp of the polynomial (23) is following:

$$Tp(\tau)=\bar{E}_0p(\tau)-E_2p^*(\tau)=(\bar{E}_0E_1-E_2\bar{E}_1)\tau+|E_0|^2-|E_2|^2.$$

Let's calculate the constants

$$\gamma_1 = Tp(0) = |E_0|^2 - |E_2|^2, \ \gamma_2 = T^2 p(0) = \left(|E_0|^2 - |E_2|^2\right)^2 - \left|\bar{E}_0 E_1 - E_2 \bar{E}_1\right|^2 .$$

Using representation $\sigma = \rho e^{i\varphi}$, we get

$$\gamma_1 = -8\left(1-\rho^2\right)^2 \left(1+\rho^2+\rho\cos\varphi\right),$$
$$\gamma_2 = 16\left(1-\rho^2\right)^2 \left(3\left(1+\rho^2\right)+4\rho\left(1+\rho^2\right)\cos\varphi - 4\rho^2\sin^2\varphi\right).$$

Taking into account inequality $0<\rho<1$, we obtain $\gamma_1 < 0$ and $\gamma_2 > 0$, therefore, by the theorem 6.8b from [8] we have that all two roots of the polynomial (23) lie in the unit disc. For example, if we get $\sigma = 0.5$ then $E_0 = \frac{11}{4}$, $E_1 = \frac{9}{2}$, $E_2 = \frac{17}{4}$ and the roots of the polynomial (23) are equal $\tau_{1,2} = \frac{-9 \pm i\sqrt{108}}{17}$, and therefore, $|\tau_{1,2}| < 1$. Thus, in this case, if $\mu\nu_1 = 0.5$ and $\mu\nu_2 = \frac{-9+i\sqrt{108}}{17}$ then the function u_1, where β,γ is a nontrivial solution of the system (22), is a non-zero solution of the homogeneous problem (1), (4).

We can continue in the same way, for example,

$$u_2(z,\bar{z}) = (1-z\bar{z})^2\left(\beta z^2 + \gamma\bar{z}^2 + \delta z\bar{z}\right), \quad |\beta|+|\gamma|+|\delta| \neq 0,$$

is a non-zero solution of homogeneous problem (1), (4) if and only if $P_5(\sigma,\tau) = 0$.

Authors tried to find concrete values of the defect numbers for the different values of σ,τ The calculation, what was done using MATHEMATICA program, showed that the defect numbers may be equal to zero and one only. Therefore, we may formulate hypothesis, what will be interesting to check:

Hypothesis. *Let's denote* $\sigma = \mu\nu_1$, $\tau = \mu\nu_2$. *The problem (1), (4) is uniquely solvable if and only if the conditions (9) hold. The condition (7) may fail at most only for one number* k*, therefore, the defect numbers of the problem are equal only to zero or one.*

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On a Dirichlet Problem for One Properly Elliptic Equation in the Unit Disk

The Dirichlet problem for the fourth order properly elliptic equation with constant coefficients in the unit disc is considered. The characteristic equation has one double root in upper half-plane and two different roots in the lower half-plane.

The solution must be found in the class of functions satisfying the Hölder condition with first order derivatives up to the boundary. The new formula for the

determination the defect numbers is obtained. The solvability conditions and the solutions of homogeneous and inhomogeneous problems are obtained in explicit form. The numerical results show that the defect numbers may be only zero and one.

Ա. Հ. Բաբայան, Մ. Հ. Մոխամմադի

Միավոր շրջանում մի ճշգրիտ էլիպսական հավասարման համար Դիրիխլեի խնդրի մասին

Դիտարկվում է միավոր շրջանում Դիրիխլեի խնդիրը հաստատուն գործակիցներով չորրորդ կարգի ճշգրիտ էլիպսական հավասարման համար: Բնութագրիչ հավասարումն ունի կրկնապատիկ արմատ վերին կիսահարթությունում և երկու տարբեր արմատներ ստորին կիսահարթությունում: Լուծումը փնտրվում է առաջին կարգի ածանցյալների հետ միասին ընդհուպ մինչև եզրը Հյոլդերի պայմանին բավարարող ֆունկցիաների դասում: Ստացվել է դեֆեկտային թվերի որոշման համար նոր բանաձևը: Դիտարկվող խնդրի լուծելիության պայմանները և համասեռ ու անհամասեռ խնդիրների լուծումները ստացվել են բացահայտ տեսքով: Թվային արդյունքները ցույց են տալիս, որ դեֆեկտային թվերը կարող են լինել միայն զրո և մեկ:

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О задаче Дирихле для одного правильно эллиптического уравнения в единичном круге

Рассматривается задача Дирихле для правильно эллиптического уравнения с постоянными коэффициентами четвертого порядка в единичном круге. Предполагается, что характеристическое уравнение имеет один двукратный корень в верхней полуплоскости и два различных корня в нижней полуплоскости. Решение ищется в классе функций, удовлетворяющих условию Гельдера вплоть до границы вместе с производными первого порядка. Получена новая формула для определения дефектных чисел. Условия разрешимости рассматриваемой задачи и решение однородной и неоднородной задач определяются в явном виде. Результаты вычислений показывают, что дефектные числа могут принимать значения только ноль или единица.

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