ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱ
НАЦИОНАЛЬНАЯ АКАДЕМИЯ НАУК АРМЕНИИ
NATIONAL ACADEMY OF SCIENCES OF ARMENIA
ДОКЛАДЫ ԶԵԿՈՒՅՑՆԵՐ REPORTS

Հատոր Том Volume 116 2016 № 2

MECHANICS

УДК 539.3

Corresponding member of NAS RA S. M. Mkhitaryan

Application of the Krylov-Bogolyubov Method for Solving Integral Equations of a Class of Contact Problems of the Theory of Elasticity

(Submitted 1/II 2016)

Keywords: *contact problems, beam bending on an elastic foundation, local deformations, integral equations.*

The method of integral equations is one of the most efficient methods for investigating contact and mixed problems of the deformable solid mechanics.Numerous results of using the method to solve such problems are reflected in [1-4]. While evolving, the theory of integral equations was constantly enriched by new analytical and numerical-analytical techniques that are quite thoroughly summarized in [5].The Krylov-Bogolyubov method for solving the Fredholm integral equations of the second kind is a well-known method used in problems of mathematical physics [6]. Its rigorous justification by methods of the functional analysis in some metric spaces is given in [7].

In the present paper, the Krylov-Bogolyubov method is used for Fredholm integral equations of the second kind with symmetric kernels that are represented by the sum of their principal parts in the form of a logarithmic function and regular parts in the form of various continuous functions. These equations describe a rather wide class of contact problems of the theory of elasticity. We will consider a contact problem for the bending of a beam of finite length on an elastic foundation, taking into account the local deformations, by the Shtaerman contact model [1]. As a result, the original integral equations are reduced to systems of linear algebraic equations of simple structures.

1. Let a beam of finite length $2a$ and height h (referred to the coordinate system 0xy) with the modulus of elasticity E_1 and Poisson`s ratio ν_1 be indented into an elastic foundation in the form of a half-plane with the corresponding elastic constants E,ν under the action of vertical

distributed forces of intensity $q(x)$. It is required to determine the pressure of the beam on the foundation, as well as the bending and shear forces in cross sections of the beam at the plane deformation. Many problems of beams and plates bending on elastic foundations within the framework of different approximate applied theories are considered in [8].

Let us derive the basic equations and relationships of the posed problem. According to the classical theory of a beam bending, the calculation of its vertical displacement $\mathrm{v}_1(x)$ in cross-section x, in case when bending moments and shear forces at the beam end sections are absent, are reduced to the solution of the following inhomogeneous Sturm-Liouville boundary value problem:

$$\begin{cases} D\dfrac{d^4\mathrm{v}_1}{dx^4} = p(x) - q(x) \quad (-a < x < a) \\ Q(x)\big|_{x=\pm a} = D\dfrac{d^3\mathrm{v}_1}{dx^3}\bigg|_{x=\pm a} = 0;\ M(x)\big|_{x=\pm a} = D\dfrac{d^2\mathrm{v}_1}{dx^2}\bigg|_{x=\pm a} = 0;\ D = E_1h^3 \big/ 12\left(1-\nu_1^2\right). \end{cases} \tag{1.1}$$

Here $p(x)$ is the required pressure of the beam on the foundation, D is the beam bending stiffness, $M(x)$ is the bending moment, and $Q(x)$ is the transversal or shear force in the cross-section with the coordinate x. Using the Green function $G(x,s)$ of the boundary value problem (1.1), its solution is represented by the formula

$$\mathrm{v}_1(x) = \int_{-a}^{a} G(x,s)\left[p(s) - q(s)\right] + \gamma x + \alpha;\quad G(x,s) = (x-s)^3 \big/ 12D \quad (-a < x < a). \tag{1.2}$$

Here the binomial $\gamma x + \alpha$ describes the rigid displacement of the beam where γ is a reduced rotation angle and α is a reduced beam settlement.

On the other hand, the vertical displacements $\mathrm{v}(x)$ of boundary points of the lower elastic half-plane are expressed by the formula [1]

$$\mathrm{v}(x) = -\vartheta\int_{-a}^{a} \ln\frac{a}{|x-s|}\, p(s)\,ds + C \quad (-\infty < x < \infty);\ \vartheta = 2\left(1-\nu^2\right)\big/\pi E. \tag{1.3}$$

In addition to these displacements caused by the global deformation of the half-plane, according to the equations of the plane theory of elasticity, the pressure acting at the given point of the surface of an elastic body should cause some additional normal displacement $\mathrm{v}_0(x)$ at this point due to the surface structure of the elastic body. According to the I. Ya. Shtaerman model [1] the local displacements $\mathrm{v}_0(x)$ are proportional to $p(x)$:

$$\mathrm{v}_0(x) = -\chi p(x) \quad (-a < x < a), \tag{1.4}$$

but outside the contact area ($|x| > a$) they are zero. Here χ is a coefficient determined experimentally; χ depends on the elastic body surface roughness. As a result, it is assumed that vertical displacements $\mathrm{v}_2(x)$ of the elastic half-plane boundary points according to (1.3)–(1.4) are determined by the formula [1]:

$$\mathrm{v}_2(x)=\mathrm{v}(x)+\mathrm{v}_0(x)=-\chi p(x)-\vartheta\int_{-a}^{a}\ln\frac{a}{|x-s|}p(s)ds+c \quad (-a<x<a). \tag{1.5}$$

Now, substituting (1.2) and (1.5) in the contact condition of two elastic bodies (in this case beam and half-plane) $\mathrm{v}_1(x)+\mathrm{v}_2(x)=\delta=const$ $(-a<x<a)$ [1], we get the governing integral equation (GIE) for $p(x)$:

$$\begin{gathered}\chi p(x)+\vartheta\int_{-a}^{a}\ln\frac{a}{|x-s|}p(s)ds+\frac{1}{12D}\int_{-a}^{a}(x-s)^3p(s)ds=\\=\frac{1}{12D}\int_{-a}^{a}(x-s)^3q(s)ds-\gamma x-\alpha \qquad (-a<x<a).\end{gathered} \tag{1.6}$$

The solution of GIE (1.6) must satisfy the conditions of the beam equilibrium

$$\int_{-a}^{a}p(s)ds=P=\int_{-a}^{a}q(s)ds; \quad \int_{-a}^{a}sp(s)ds=M=\int_{-a}^{a}sq(s)ds. \tag{1.7}$$

After solving the GIE (1.6)–(1.7), according to (1.2) we have for moments $M(x)$ and forces $Q(x)$ in the beam cross section with coordinate x:

$$\begin{gathered}M(x)=D\frac{d^2\mathrm{v}_1}{dx^2}=\frac{1}{2}\int_{-a}^{a}|x-s|p(s)ds-\frac{1}{2}\int_{-a}^{a}|x-s|q(s)ds;\\ Q(x)=D\frac{d^3\mathrm{v}_1}{dx^3}=\frac{1}{2}\int_{-a}^{a}\operatorname{sign}(x-s)p(s)ds-\frac{1}{2}\int_{-a}^{a}\operatorname{sign}(x-s)q(s)ds\end{gathered} \quad (-a\le x\le a) \tag{1.8}$$

Further we pass to the dimensionless quantities in (1.6)–(1.8)

$$\xi=x/a;\ \eta=s/a;\ p_0(\xi)=p(a\xi)/E;\ q_0(\xi)=q(a\xi)/E;$$

$$\vartheta_0=a\vartheta/\chi;\ \lambda_0=a^4/12\chi D;\ \gamma_0=a\gamma/\chi E;\ \ \alpha_0=\alpha/\chi E.$$

Then the GIE (1.6) becomes

$$\begin{gathered}p_0(\xi)+\int_{-1}^{1}\left[\vartheta_0\ln\frac{1}{|\xi-\eta|}+\lambda_0G_0(\xi,\eta)\right]p_0(\eta)d\eta=h_0(\xi)-\gamma_0\xi-\alpha_0,\\ G_0(\xi,\eta)=|\xi-\eta|^3,\ h_0(\xi)=\lambda_0\int_{-1}^{1}G_0(\xi,\eta)q_0(\eta)d\eta,\end{gathered} \tag{1.9}$$

beam equilibrium condition (1.7) pass to

$$\int_{-1}^{1}p_0(\eta)d\eta=P_0 \quad (P_0=P/aE); \quad \int_{-1}^{1}\eta p_0(\eta)d\eta=M_0 \quad \left(M_0=M/a^2E\right) \tag{1.10}$$

and the relationship (1.8) to

$$M_0(\xi)=\frac{1}{2}\int_{-1}^{1}|\xi-\eta|p_0(\eta)d\eta-f_0(\xi);\quad M_0(\xi)=M(a\xi)/a^2E$$
$$Q_0(\xi)=\frac{1}{2}\int_{-1}^{1}\operatorname{sign}(\xi-\eta)p_0(\eta)d\eta-g_0(\xi);\quad Q_0(\xi)=Q(a\xi)/aE \qquad (-1\le\xi\le1) \qquad (1.11)$$
$$f_0(\xi)=\frac{1}{2}\int_{-1}^{1}|\xi-\eta|q_0(\eta)d\eta;\quad g_0(\xi)=\frac{1}{2}\int_{-1}^{1}\operatorname{sign}(\xi-\eta)q_0(\eta)d\eta.$$

The formulas (1.9)-(1.11) represent the basic equations and relationships of the posed problem. These equations but with different $G_0(\xi,\eta)$ describe similar contact problems for foundations in the form of strips, wedges and other areas.

2. From a practical point of view, the determination of the contact pressure $p_0(\xi)$ not only within the contact area $-1<\xi<1$ but also at its ends $\xi=\pm1$ is of interest. Therefore,we represent the solution of the GIE (1.9)–(1.10) in the form of

$$p_0(\xi)=A_0\xi+B_0+\sqrt{1-\xi^2}\chi_0(\xi) \quad (-1\le\xi\le1) \qquad (2.1)$$

where A_0 and B_0 are unknown coefficients, $\chi_0(\xi)$ is an unknown function continuous in $[-1,1]$. Substituting $\xi=\pm1$ in (2.1), we come to simple equations from which we get

$$A_0=\frac{1}{2}\left[p_0(1)-p_0(-1)\right];\ \ B_0=\frac{1}{2}\left[p(1)+p(-1)\right], \qquad (2.2)$$

i.e. after determination A_0 and B_0, values $p_0(\pm1)$ will be found from (2.2).

Now substituting the expression $p_0(\xi)$ from (2.1) in the GIE (1.9) we get

$$A_0\xi+B_0+\sqrt{1-\xi^2}\chi_0(\xi)+A_0\vartheta_0I_2(\xi)+B_0\vartheta_0I_1(\xi)+A_0\lambda_0\int_{-1}^{1}G_0(\xi,\eta)\eta d\eta+$$
$$+B_0\lambda_0\int_{-1}^{1}G_0(\xi,\eta)d\eta+\int_{-1}^{1}\left[\vartheta_0\ln\frac{1}{|\xi-\eta|}+\lambda_0G_0(\xi,\eta)\right]\sqrt{1-\eta^2}\chi_0(\eta)d\eta= \qquad (2.3)$$
$$=h_0(\xi)-\gamma_0\xi-\alpha_0;\quad I_1(\xi)=\int_{-1}^{1}\ln\frac{1}{|\xi-\eta|}d\eta;\quad I_2(\xi)=\int_{-1}^{1}\eta\ln\frac{1}{|\xi-\eta|}d\eta \quad (-1\le\xi\le1).$$

The computation of these elementary integrals gives

$$I_1(\xi)=2-(1-\xi)\ln(1-\xi)-(1+\xi)\ln(1+\xi);$$
$$I_2(\xi)=\frac{1}{2}\left[(1+\xi)^2\ln(1+\xi)-(1-\xi)^2\ln(1-\xi)-2\xi\right]+\xi I_1(\xi) \qquad (-1\le\xi\le1) \quad (2.4)$$

whence

$$I_1(-1)=I_1(1)=2-\ln4;\quad I_2(-1)=-I_2(1)=-1. \qquad (2.5)$$

The substitution of (2.1) into (1.10) results in

$$\frac{2}{3}A_0+\int_{-1}^{1}\eta\sqrt{1-\eta^2}\chi_0(\eta)d\eta=M_0;\quad 2B_0+\int_{-1}^{1}\sqrt{1-\eta^2}\chi_0(\eta)d\eta=P_0. \qquad (2.6)$$

We now turn to solving GIE (1.9)–(1.10) or equivalent tothem equations (2.3)–(2.4) and (2.6) by Krylov-Bogolyubov method. First, let us calculate the integral

$$I(\xi)=\int_{-1}^{1}\ln\frac{1}{|\xi-\eta|}\sqrt{1-\eta^2}\chi_0(\eta)d\eta \qquad (-1\le\xi\le1).$$

For this purpose we divide the segment $-1\le\eta\le1$ by points $\eta_k=-1+2k/N$ $\left(k=\overline{1,N-1}\right)$ into N equal partial intervals $\eta_{k-1}<\eta<\eta_k$ $\left(k=\overline{1,N};\eta_0=-1,\eta_N=1\right)$.

On each such partial interval we assume that the function $X_0(\eta)=\sqrt{1-\eta^2}\chi_0(\eta)$ takes on a constant value X_k, i.e. we replace the continuous function $X_0(\eta)$ with a step function. Further compute the integral $J(\xi)$ at points $\xi_j=-1+(2j-1)/N$ $\left(j=\overline{1,N}\right)$ which are midpoints of partial intervals $\left(\eta_{j-1},\eta_j\right)$. Then

$$J\left(\xi_j\right)=\sum_{k=1}^{N}L_{jk}X_k;\; L_{jk}=-\int_{-1+2(k-1)/N}^{-1+2k/N}\ln\left(\frac{2j-1}{N}-1-\eta\right)d\eta \quad \left(j,k=\overline{1,N}\right). \quad (2.7)$$

The integral L_{jk} from (2.7) are calculated accurately by considering separately the cases $k\le j-1,\ k\ge j+1$ and $k=j$. As a result we get

$$L_{jk}=\frac{2|k-j|-1}{N}\ln\left(\frac{\big|2|k-j|-1\big|}{N}\right)-\frac{2|k-j|+1}{N}\ln\left(\frac{2|k-j|+1}{N}\right)+\frac{2}{N} \quad (k,j=1,N). \quad (2.8)$$

Using the above method we compute also the following integrals at nodal points ξ_j

$$N(\xi)=\int_{-1}^{1}G_0(\xi,\eta)\sqrt{1-\eta^2}\chi_0(\eta)d\eta,\quad S_0(\xi)=\int_{-1}^{1}G_0(\xi,\eta)d\eta,\quad Q_0(\xi)=\int_{-1}^{1}G_0(\xi,\eta)\eta d\eta.$$

We have

$$N\left(\xi_j\right)=\frac{2}{N}\sum_{k=1}^{N}G_0\left(\xi_j,\xi_k\right)X_k,S\left(\xi_j\right)=\frac{2}{N}\sum_{k=1}^{N}G_0\left(\xi_j,\xi_k\right),Q\left(\xi_j\right)=\frac{2}{N}\sum_{k=1}^{N}\xi_kG_0\left(\xi_j,\xi_k\right). \quad (2.9)$$

Now we require that the integral equation (2.3) will be satisfied at nodal points ξ_j. Then we obtain the following system of linear algebraic equations (SLAE) for unknown variables $A_0,B_0,\gamma_0,\alpha_0$ and X_j:

$$\begin{aligned}&\left[\xi_j+\vartheta_0I_2\left(\xi_j\right)+\lambda_0Q_0\left(\xi_j\right)\right]A_0+\left[1+\vartheta_0I_1\left(\xi_j\right)+\lambda_0S_0\left(\xi_j\right)\right]B_0+\gamma_0\xi_j+\alpha_0+X_j+\\&+\sum_{k=1}^{N}\left[\vartheta_0L_{jk}+\frac{2\lambda_0}{N}G_0\left(\xi_j,\xi_k\right)\right]X_k=h_0\left(\xi_j\right) \quad \left(j=\overline{1,N};\quad \xi_j=-1+(2j-1)/N\right)\end{aligned} \quad (2.10)$$

where $I_p\left(\xi_j\right)$ $(p=1,2)$ are calculated by the formula (2.4), L_{jk}, $S_0\left(\xi_j\right)$ and $Q_1\left(\xi_j\right)$ are given by (2.8)–(2-9). The SLAE (2.10) is completed by two equations

$$2B_0+\frac{2}{N}\sum_{k=1}^{N}X_k=P_0;\qquad \frac{2}{3}A_0+\frac{2}{N}\sum_{k=1}^{N}\xi_kX_k=M_0 \quad (2.11)$$

obtained from the equilibrium conditions (1.10) and by two equations obtained from (2.3) for $\xi = \pm 1$. The following desired integrals are calculatedin the process:

$$H_0^{\pm} = \int_{-1}^{1} G_0(\pm 1, \eta)\, d\eta = \frac{2}{N}\sum_{k=1}^{N} G_0(\pm 1, \xi_k);\; L_0^{\pm} = \int_{-1}^{1} G_0(\pm 1, \eta)\eta\, d\eta = \frac{2}{N}\sum_{k=1}^{N} \xi_k G_0(\pm 1, \xi_k);$$

$$\int_{-1}^{1} G(\pm 1, \eta)\sqrt{1-\eta^2}\chi_0(\eta)\, d\eta = \frac{2}{N}\sum_{k=1}^{N} G_0(\pm 1, \xi_k) X_k;\quad \int_{-1}^{1} \ln(1 \pm \eta)\sqrt{1-\eta^2}\chi_0(\eta)\, d\eta =$$

$$= \begin{cases} \dfrac{2}{N}\sum\limits_{k=1}^{N}\left\{ k\ln\left(\dfrac{2k}{N}\right) - (k-1)\ln\left[\dfrac{2(k-1)}{N}\right] - 1 \right\} X_k; \\ 2\sum\limits_{k=1}^{N}\left\{ \left[1 - \dfrac{k-1}{N}\right]\ln\left[2 - \dfrac{2(k-1)}{N}\right] - \left(1 - \dfrac{k}{N}\right)\ln\left(2 - \dfrac{2k}{N}\right) - \dfrac{1}{N} \right\} X_k \end{cases}. \quad (2.12)$$

We enter new unknowns $Y_1 = A_0,\ Y_2 = B_0,\ Y_3 = \gamma_0,\ Y_4 = \alpha_0,\ Y_{k+4} = X_k$ $\left(k = \overline{1, N}\right)$, and combine SLAE (2.10) with (2.11) and two equations obtained from (2.3) for $\xi = \pm 1$. Taking into account (2.12), we finally obtain the following SLAE in canonical form:

$$Y_j + \sum_{k=1}^{N+4} R_{jk} Y_k = a_j \quad \left(j = \overline{1, N+4},\right), \qquad (2.13)$$

where the number of equations and unknowns are equal to $N+4$. In (2.13) the following notations are made:

$$R_{1k} = \begin{cases} 0 \ \left(k = \overline{1,4}\right);\ a_1 = \dfrac{3}{2} M_0; \\ \dfrac{3}{N}\,\xi_{k-4}\left(k = \overline{5, N+4}\right); \end{cases} \quad R_{2k} = \begin{cases} 0 \ \left(k = \overline{1,4}\right);\ a_2 = \dfrac{1}{2} P_0; \\ \dfrac{1}{N}\,\xi_{k-4}\left(k = \overline{5, N+4}\right); \end{cases}$$

$$R_{3k} = \begin{cases} 1 - \vartheta_0 I_2(-1) - \lambda_0 L_0^- \qquad (k=1); \\ -\left[1 + \vartheta_0 I_1(-1) + \lambda_0 H_0^-\right] \quad (k=2); \quad a_3 = -h_0(-1) \\ 0 \quad (k=3); \qquad -1 \quad (k=4); \\ \dfrac{2}{N}\left\langle \vartheta_0 \left\{ k^* \ln\left(\dfrac{2k^*}{N}\right) - (k^*-1)\ln\left[\dfrac{2(k^*-1)}{N}\right] - 1 \right\} - \right. \\ \left. -\lambda_0 G_0(-1, \xi_k^*) \right\rangle \ \left(k^* = k-4;\ \xi_k^* = \xi_{k-4}\right); \quad k = \overline{5, N+4} \end{cases}$$

$$R_{4k} = \begin{cases} 1 + \vartheta_0 I_2(1) + \lambda_0 L_0^+ \qquad (k=1); \\ 1 + \vartheta_0 I_1(1) + \lambda_0 H_0^+ \quad (k=2); \quad a_4 = h_0(1) \\ 1 \quad (k=3); \quad 0 \quad (k=4); \\ 2\vartheta_0 \left\{ \left(1 - \dfrac{k^*}{N}\right)\ln\left(2 - \dfrac{2k^*}{N}\right) - \left[1 - \dfrac{k^*-1}{N}\right]\ln\left[2 - \dfrac{2(k^*-1)}{N}\right] + \right. \\ \left. + \dfrac{1}{N} \right\} + \dfrac{2\lambda_0}{N} G_0(1, \xi_k^*) \quad k = \overline{5, N+4}; \quad k^* = k-4;\ \xi_k^* = \xi_{k-4} \end{cases}$$

$$R_{jk} = \begin{cases} \xi_j^* + \vartheta_0 I_2(\xi_j^*) + \lambda_0 Q_0(\xi_j^*) & (k=1); \quad (\xi_j^* = \xi_{j-4},\ \ j = 5,6,\dots,N+4); \\ 1 + \vartheta_0 I_1(\xi_j^*) + \lambda_0 S_0(\xi_j^*) & (k=2); \qquad a_j = h_0(\xi_j^*) \\ \xi_j^* & (k=3); \quad 1 \quad (k=4); \\ \vartheta_0 L_{jk}^* + \dfrac{2\lambda_0}{N} G_0(\xi_j^*, \xi_k^1); & L_{jk}^* = L_{j-4,k-4} \quad (j,k = \overline{5, N+4}). \end{cases}$$

The values $I_p(\pm 1)$ $(p = 1, 2)$ in R_{3k} and R_{4k} are given by formulas (2.5).

Now, express the main characteristics of the problem: the dimensionless contact pressure $p_0(\xi)$ from (2.1), the dimensionless bending moments $M_0(\xi)$ and transverse forces $Q_0(\xi)$ from (1.11), in terms of the solution of SLAE (2.13).

After calculating the elementary integrals the values of these characteristics on the partial intervals (η_{j-1}, η_j) $(j = \overline{1, N})$ will be given by the formulas

$$p_0(\xi_j) = Y_1 \xi_j + Y_2 + Y_{j+4}; \quad M_0(\xi_j) = \frac{Y_1}{2}\left[\xi_j(\xi_j^2 - 1) + \frac{2}{3}(1 - \xi_j^3)\right] + \frac{Y_2}{2}(1 + \xi_j^2) +$$

$$+ \frac{1}{4}\sum_{k=1}^{N}\left[2\xi_j(2\xi_j - \xi_k - \xi_{k-1}) + \xi_k^2 + \xi_{k-1}^2 - 2\xi_j^2\right] Y_{k+4} - f_0(\xi_j);$$

$$Q_0(\xi_j) = \frac{Y_1}{2}(\xi_j^2 - 1) + Y_2 \xi_j + \frac{1}{2}\sum_{k=1}^{N}(2\xi_j - \xi_k - \xi_{k-1}) Y_{k+4} - g_1(\xi_j);$$

$$f_0(\xi_j) = \frac{1}{N}\sum_{k=1}^{N} |\xi_j - \xi_k| q_k; \quad g_0(\xi_j) = \frac{1}{N}\sum_{k=1}^{N} \operatorname{sign}(\xi_j - \xi_k) q_k; \ \xi_j = -1 + (2j - 1)/N \quad (j = \overline{1, N}).$$

Here q_k are accepted constant values of the function $q_0(\eta)$ on the partial interval (η_{k-1}, η_k) $(k = \overline{1, N};\ \eta_0 = -1,\ \eta_N = 1)$.

Thus, after solving the SLAE (2.13) the main characteristics of the posed problem are defined by very simple formulas (2.14). In the course of solving (2.13) the quantities γ_0, α_0 are determined.

Institute of Mechanics of NAS RA
smkhitaryan 39@rambler.ru

Corresponding member of NAS RA S. M. Mkhitaryan

Application of the Krylov-Bogolyubov Method for Solving Integral Equations of a Class of Contact Problems of the Theory of Elasticity

The well-known Krylov-Bogolyubov method is used in solving Fredholm integral equations of the second kind, with symmetric kernels represented by the sum of the logarithmic function and different continuous functions. These equations describe a rather wide class of contact problems of the theory of elasticity, in particular,

thessscontact problem on the bending of a beam on an elastic half-plane within the framework of the Shtaerman contact model, discussed in the paper.

ՀՀ ԳԱԱ թղթակից անդամ Ս. Մ. Մխիթարյան

Կրիլովի և Բոգոլյուբովի մեթոդի կիրառությունը առաձգականության տեսության կոնտակտային խնդիրների մի դասի ինտեգրալ հավասարումների լուծմանը

Կրիլովի և Բոգոլյուբովի հայտնի մեթոդը կիրառվում է սիմետրիկ կորիզներով, որոնք ներկայացվում են լոգարիթմական ֆունկցիայի և տարբեր անընդհատ ֆունկցիաների գումարով: Այդ հավասարումներով նկարագրվում է առաձգականության տեսության կոնտակտային խնդիրների բավականաչափ լայն դաս, մասնավորապես, Ի. Յա. Շտաերմանի կոնտակտի մոդելի շրջանակներում հոդվածում դիտարկված առաձգական կիսահարթության վրա հեծանի ծռման կոնտակտային խնդիրը:

Член-корреспондент НАН РА С. М. Мхитарян

Применение метода Крылова и Боголюбова к решению интегральных уравнений одного класса контактных задач теории упругости

К решению интегральных уравнений Фредгольма второго рода с симметрическими ядрами, представленными суммами логарифмической функции и различных непрерывных функций, применяется известный метод Крылова и Боголюбова. Этими уравнениями описывается достаточно широкий класс контактных задач теории упругости, в частности, рассматриваемая в статье контактная задача об изгибе балки на упугой полуплоскости в рамках модели контакта И.Я. Штаермана.

References

1. *Shtaerman I.Y.* – M.–L. Gostekhizdat. 1949. 270 p. (in Russian).
2. *Galin L.A.* –M. Nauka. 1980. 304 p. (in Russian)
3. *Vorovich I. I., Aleksandrov V. M., Babeshko V. A.* –M. Nauka. 1974. 456 p.(in Russian).
4. Development of the theory of contactproblems in the USSR. M. Nauka. 1976. 493 p.
5. *Verlan A.F. Sizikov V.S.* – Handbook. Kiev. Naukova dumka. 1986. 544 p.(in Russian).
6. *Kantorovich L. V., Krylov V. I.* – M.-L. 1962. 708p. (in Russian).
7. *Gabdulkhaev B. G.* – Izvestie. Vuzov. 2002. Math. №10. (485). P.34-47. (in Russian).
8. *Vlasov V. Z., Leontiev N. N.* – M. Fizmatgiz. 1960. 491 p. (in Russian).