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Stresses in a Tapered Space Elevator Tube

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Keywords: *space elevator, tapered hollow tube, mechanical stresses.*

Introduction. The space elevator is a new space transportation engineering structure rising from the Earth's equator surface to above geostationary orbit extended out to the level of 144,000 km where it terminates in a counterweight. The space elevator structure would permit to travel into space from the Earth's surface without the use of space rockets. Conditioned by resulting force of the Earth gravity inward force and centrifugal outward force caused by the Earth's daily spinning the structure would be held in tension in which the maximum tensile strength would reach up to 120 GPa. Currently available materials do not meet this requirement. Advances in carbon nanotube technology could make it possible to create a strong type of carbon nanotube materials in future with tensile strength of up to 200 GPa [1,2]. These carbon nanotubes with high tensile strength, combined with relatively low density, make nanotubes an excellent construction materialfor a space elevator. The mathematical and engineering concepts of the space elevator were first introduced in [3-5], where problems of buckling, strength and vibration were discussed. Reviews, developments of the space elevator and detailed design for construction and operation are given, particularly, in [6-15].

The paper is devoted to a new simple design model of linearly tapered space elevator hollow tube.

Model statement. Let us consider a hollow tapered thin tube with constant internal radius r_0 and variable thickness $h(\gamma)=h_0+\gamma\delta,\quad r_0 >> h(\gamma)$ linearly varyingalong tube height γ (Fig 1).

Introducing dimensionless coordinate $x=\gamma/R_0$, (γ is the distance of a point on the tube measured from the ground level), for thin tapered elevator tube the cross sectional area $S(x)$ varying a with position along the tube can be written as

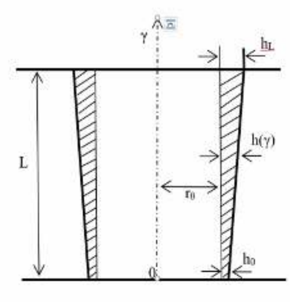

Fig 1. The tapered hollow tube.

$$S(x) = S_0(1 + ax), \qquad S_0 = 2\pi r_o h_0, \tag{1}$$

where S_0 is the cross sectional area of the tube at the ground level.

The stress in tube $\sigma_0(x)$ is determined from the equation [9,10]

$$\frac{d\left[S(x)\sigma_0(x)\right]}{dx} - R_0 \rho g_0 S(x) g(x) = 0, \qquad g(x) = (1+x)^{-2} - \beta(1+x); \tag{2}$$

where R_0 is the Earth's radius, ρ the mass density of elevator tube, g_0 is the acceleration of gravity at Earth's surface $\beta = Rg_0^{-1}\omega^2 = 1/288$, ω is the Earth's rotational angular velocity.

The dimensionless stress $\sigma(x) = (R_0\rho g_0)^{-1}\sigma_0(x)$ under condition $\sigma(0) = 0$ can be calculated as[10]

$$\sigma(x) = \frac{\int_0^x g(z)S(z)dz}{S(x)}; \tag{3}$$

The minimal characterizing length L of the tube will be determined from condition

$$\int_0^L g(z)S(z)dz = 0. \tag{4}$$

In this way we can finally find that

$$\sigma(x) = \frac{x\left[3\big(574 - x(3+x)\big) - a\big(1728 + x(1+x)(3+2x)\big)\right]}{1728(1+x)(1+ax)} + \frac{a\ln(1+x)}{(1+ax)} \tag{5}$$

Let note that in limiting case $a \to \infty$ we have that

$$\sigma_\infty(x) = \frac{-x\left(1728 + 3x + 5x^2 + 2x^3\right) + 1728(1+x)\ln(1+x)}{1728x(1+x)}.$$

Analysis and numerical results. The negative values of parameter a correspond to the tube elevator narrowing up to the upper end. On the Fig.2 are presented the curves describing the stress states of the space elevator. The upper curve corresponds to the tapered tube with $a = -0.01$, the middle curve corresponds to the non-tapered tube with $a = 0$, the lower curve corresponds to the tube narrowing at ground level (widening at upper level) with $a = 0.01$.

Since for the space elevator narrowing at the upper end the stresses exceeding maximal stress of non-tapered tube only the tapered tubes with positive values of $a > 0$ must be considered (see Fig 2).

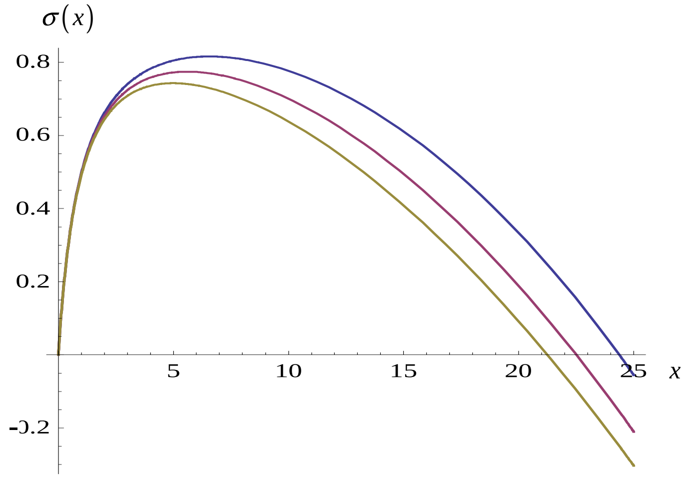

Fig 2. The stress states for three different space elevator tubes.

In Table 1 for different values of parameter a the data are presented for the maximum tensile stresses $\sigma_* = \max \sigma(x)$, the tube minimal length L_0 under which all stresses are positive (tensile), as well, the taper ratio $\beta = \max S(x)/\min S(x) = S(L_0)/S_0$. Point x_0 is the level at the tube, where the maximal stress occurs. For non tapered cable the maximum tensile stress in the cable occurs at geostationary level [3, 4].

Table 1

a	σ_*	x_0	L_0	β
-0.01	0.81	6.21	23.92	0.76
0	**0.78**	**5.62**	**22.54**	**1**
0.05	0.65	3.78	18.38	1.92
0.1	0.59	3.06	16.66	2.70
0.2	0.53	2.68	14.85	3.97
0.5	0.42	1.81	12.93	7.47
1	0.35	1.61	11.88	12.88
1.5	0.32	1.42	11.46	18.19
2	0.27	1.39	11.32	23.64

From the results of Table 1 and Fig. 2 data it follows that increasing the taper ratio β brings to the sufficient decreasing of stresses and limiting lengths compared with non tapered cable.

Conclusions. The new design model for a space elevator cable is presented, namely, the linearly tapered and widening at upper level hollow tube, where the maximum stress and limiting length can be sufficiently reduced. For these reasons this model can be useful in constructions of a space elevator cable.

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Stresses in a Tapered Space Elevator Tube

A new design model of the tapered hollow tube is presented, which can be useful in the space elevator cable constructions.

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Напряжения в конической трубке космического лифта

Предложена новая модель конической полой трубки, которая может быть полезна в конструкциях троса космического лифта.

Ակադեմիկոս Ս. Ա. Համբարձումյան, Մ. Վ. Բելուբեկյան,
Կ. Բ. Ղազարյան

Լարումներըտիեզերականվերելակիկոնաձևխողովակում

Առաջարկված է կոնաձև սնամեջ խողովակի նոր մոդել, որը կարող է օգտակար լինել տիեզերական վերելակի ճոպանի կառուցվածքներում:

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