ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱ
НАЦИОНАЛЬНАЯ АКАДЕМИЯ НАУК АРМЕНИИ
NATIONAL ACADEMY OF SCIENCES OF ARMENIA
ДОКЛАДЫ ԶԵԿՈՒՅՑՆԵՐ REPORTS

Հատոր Том Volume 115 2015 № 2

MECHANICS

УДК 539.3

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The Boundary Problem Molelling of Rough Surfaces Continuous Media with Coupled Physicomechanical Fields*

(Submitted by academician S.A. Ambartsumian 17/IV 2015)

Keywords: *contact of bodies, modelling of boundary value problems, rough surface, physical and mechanical fields, heterogeneous material.*

1. Introduction. Mathematical modelling is the basis of the study of most modern problems in physics and development of many modern innovative technologies. Among other research analyses requiring the application of mathematical modelling methods is the analysis of different surface contacts in layered structures. In the context of coupled physicomechanical fields this problem becomes particularly important with relation to the studies of dynamic processes in the composite joints with rough surfaces.

Back in 1882 Hertz solved the problem of contact between two elastic bodies with curved surfaces [1]. This classical result had the mechanics of contact interaction for about a century, before Johnson, Kendall and Roberts are founded a similar solution for adhesive contact [2] (JKR - theory). Further progress in mechanics of contact interaction in the mid-20th century is associated with Bowden and Tabor [3]. They were the first to identify the importance of accounting forthe surface roughness of bodies in contact. Roughness causes the true contact area between the sliding bodies, which is a much smaller than the apparent contact area. These views changed the direction of many studies in tribology significantly, and contributed towards a number of theories of contact mechanics. Pioneering works in this field are the works of Archard [4], which concluded that when the elastic contact of rough surfaces, the contact area is approximately proportional to the normal force. Further important contributions to the theory of rough surfaces made Greenwood and Williamson [5] and Persson [6]. The main result of these works is the proof that the actual area of contact of rough surfaces in a rough approximation,

*I wish to express special gratitude to my teacher Pr. Mels Belubekyan and dedicate this research to his eightieth anniversary.

proportional to the normal force, whiles the characteristics of a certain micro contact (pressure, the size of micro contact) weakly depends on the load.

In the basis of mathematical modelling there have always been physical or geometric hypothetical conceptions of some components of physicomechanical fields. To model new mathematical boundary value problems in mechanics of deformable solid,considering the nature of the quantities describing physical and mechanical fields in thin-walled elements various hypotheses on distributions of characteristic variables of the stress strain fields were introduced [7]-[10]. An analogical approach was successfully appliedto-electromagnetoelastic fields, thus developing the theory of electromagneto-elasticity of thin-walled structural elements [11].

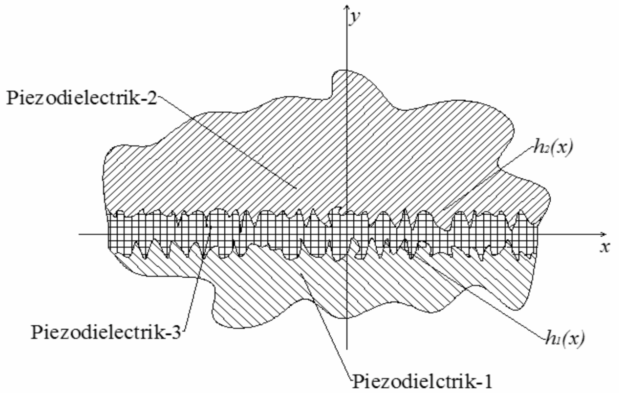

Fig.1. Contact of rough surfaces in three-layer piezodielectric composite.

To simulate the effects associated with the influence of roughness on the amplitude coefficients of reflection and transmission at the boundary between two dielectric media with different optical parameters, it is proposed to use two new parameters: large-scale and small-scale RMS roughness [12]. The problem of contact interaction simulation with pressed to each other semi-infinite elastic rough bodies, and characterized by two widely different typical length scales for theroughness [13], is solved by the method of successive approximations.

The solution to the problem, which belongs to the class of free boundary problems, is obtained by calculating the Green's function, which relates the pressure distribution with the normal displacements on the boundary [14]. The problem is then formulated as Feedhole’s integral equation of the first kind with logarithmic kernel. When two bodiesare separated by a small distance of surface roughness, adhesion and friction begin to play an important role in their interaction. The evaluation of the effect of roughness becomes extremely difficult when the roughness is comparable to the distance between the bodies.

In the paper [15] the current status of the problem is described and the problem with the introduction of dispersion forces of physical origin is explored. It was experimentally shown additives of forces introduced with the effect of roughness on the interaction between bodies. In [16], by calculating Green's function, which relates the pressure distribution in normal displacements at the interface to the adhesive contact, the authors conducted an analysis of adhesive rough surface contact. In another work [17] studied the influence of surface roughness on the entered modelled dispersion forces. The analysis is carried out opinions on crucial issues.

By introduction of surface-exponential functions (Surface Exponential Function-SEF), the present article formulates new hypotheses about the distributions of the relevant characteristic of physicomechanical values (hypotheses MELS) depending on the model of docking of rough-boundary environments with different coupled physicomechanical fields.

2. Model contacts adjacent to half-spaces with rough surfaces. Let's consider a dynamic process at the junctions of the two half-spaces with rough surfaces $y = h_1(x)$ and $y = h_2(x)$ respectively (Fig.1).

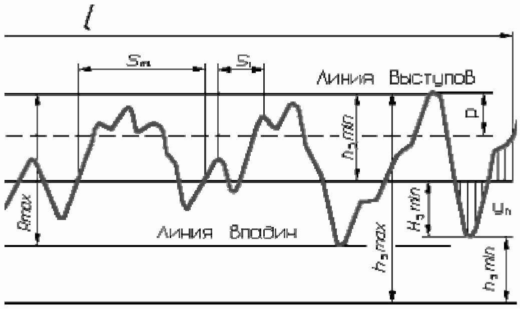

Fig. 2. Roughness contour and the basic roughness parameters: l -base line, m -centre line, S_{im} -the average pitch of the irregularities of the profile, $R_{\max}$ -maximum height of the profile.

The coordinate system xoy is chosen so that the surface is perpendicular to the parallel roughness, and the axis oz is parallel to these lines.The connection via elastic rough surfaces always forms a three-layer composite. Different methods of layer connection lead naturally to the formation of different three-layer packets on the surface zone of the compounds. A set of random function $y = h_m(x);\ m = 1;2$, which defines the main parameters of roughness (Fig. 2) plays a decisive role in connecting surfaces. The roughness contour depicts the number, shape and size of the projections and depressions of the irregularities. Considering the fact that the height of the projections and depressions of the surface roughness ranges from 0.08 to 500 microns or more, and the ratio of the average step roughness on the height profile is of one hundred $S_{im} \sim 100 \cdot R_{\max}$, it can be assumed that $h_m(x) \in L_1\{\mathbb{R}\}$ in relation to the functions of roughness.

Generally, the materials of the adjacent environments can have different properties and can be described by different thermodynamic material ratios. Reservations, regarding the nature of the coupling of physical fields or the linearity/nonlinearity of the thermodynamic material ratios in each composite environment, will be addressedupon the necessity in certain tasks.

Firstly, we'll consider the case of pasted half-spaces, from which by producingof material permanent adhesive environment to zero, we get the problem of rough surfaces with a vacuum gap of variable thickness.

Model-1 Two elastic deformable half-space made of materials characterized by associated physical fields (electro-elasticity, magneto-elasticity, thermos-elasticity, etc.) interconnected by glue with relevant physical and mechanical characteristics. Then in each half-space

$$\Omega_1 = \{|x| < \infty, -\infty < y \le h_1(x), |z| < \infty\}$$

and

$$\Omega_2 = \{|x| < \infty, h_2(x) \le y < \infty, |z| < \infty\}$$

the corresponding equations of the environment are solved, $L_{1n}[u_i, \varphi, \psi, \vartheta] = 0; \quad n \in \{1; 2; ...\}$ and $L_{2n}[u_i, \varphi, \psi, \vartheta] = 0; \quad n \in \{1; 2; ...\}$ taking into account thermodynamic material ratios of this environment: $\hat{\sigma}(u_i, \varphi, \psi, \vartheta)$ - thermo-mechanical stress, $\vec{\mathrm{D}}(u_i, \varphi, \psi, \vartheta)$ - displacement electric field, $\vec{\mathrm{B}}(u_i, \varphi, \psi, \vartheta)$ magnetic field, $\theta(u_i, \varphi, \psi, \vartheta)$ - temperature. The number of equations $n \in \{1; 2; ...\}$ and unknown variables naturally depend on the associated physical and mechanical fields.

In the inner adhesive slit $\Omega_3 = \{|x| < \infty, h_1(x) \le y \le h_2(x), |z| < \infty\}$ of variable width $\xi(x) = |h_2(x) - h_1(x)|$, equations for an adhesive glue material $L_{3n}[u_i, \varphi, \psi, \vartheta] = 0; \quad n \in \{1; 2; ...\}$, with corresponding thermodynamic material ratio, are solved.

The three sets of equations mentioned above – $L_{\alpha n}[u_i, \varphi, \psi, \vartheta] = 0; \quad n \in \{1; 2; ...\}$ and boundary conditions on each surface $y = h_m(x)$, with varying normal $\vec{n}^{(m)}(x) = \{n_j^{(m)}(x, h_m(x))\}$ where $m = 1; 2.$,

$$n_j^{(m)}(x, h_m(x)) = \left\{ \frac{h'_m(x)}{\sqrt{1 + [h'_m(x)]^2}}; \frac{1}{\sqrt{1 + [h'_m(x)]^2}}; 0 \right\},$$

together with the terms of the attenuation of the relevant physicomechanical fields at infinity form a mathematical boundary task in the three-layer composite with rough surfaces in contact.

Considering the fact that in linear theories of mechanics related to physicomechanical fields all the connections between physical fields are identical, the material ratio could formally be recorded in the same form: $\sigma_{ij} = c_{ijnm} u_{n,m} + \alpha_{mij} \phi_{,m}$, $\quad S_m = \alpha_{mij} u_{i,j} - \beta_{mn} \phi_{,n}$, where except for the known mechanical stresses σ_{ij}, the elastic deformation tensor $u_{i,j}$ and the elastic constants $\hat{c} \equiv \left(c_{ijmn}\right)_{6\times 6}$, the generalized expression for the material thermodynamic relations is shownthrough the signs of tensors of physical constants $\alpha_{mij} \in \left\{e_{mij}; d_{mij}; \lambda_{mij}\right\}$ and $\beta_{mn} \in \left\{\varepsilon_{mn}; \mu_{mn}; g_{mn}\right\}$, the components defining characteristics $S_m\left(x_j,t\right) \in \left\{D_m\left(x_j,t\right); B_m\left(x_j,t\right); \theta\left(x_j,t\right)\right\}$ and potentials $\phi\left(x_j,t\right) \in \left\{\varphi\left(x_j,t\right); \psi\left(x_j,t\right); \vartheta\left(x_j,t\right)\right\}$ of the corresponding physical fields , and coordinate system $\left\{x,y,z\right\} \triangleq \left\{x_1,x_2,x_3\right\}$.

To avoidcumbersome formulas, we will assume that the materials adjacent half-spaces (thick plates) as well as the thin adhesive layer are piezoelectric crystals of hexagonal symmetry class $6mm$, which are polarized along the axis $oz \| \bar{p}$.

In this case, the boundary conditions of the linear electro-elasticity and the conditions of decay at infinity, with the indices $i;j \in \left\{1;2;3\right\}$ and $m \in \left\{1,2\right\}$, have the following form:

$$\left(\sigma_{ij}^{(m)}(x,h_m(x),t) - \sigma_{ij}^{(3)}(x,h_m(x),t)\right) \cdot n_j^{(m)}\left(x,h_m(x)\right) = 0 \tag{2.2}$$

$$\left(D_j^{(m)}(x,h_m(x),t) - D_j^{(3)}(x,h_m(x),t)\right) \cdot n_j^{(m)}\left(x,h_m(x)\right) = 0 \tag{2.3}$$

$$w_i^{(m)}(x,h_m(x),t) = w_i^{(3)}(x,h_m(x),t) \tag{2.4}$$

$$\varphi_m(x,h_m(x),t) = \varphi_3(x,h_m(x),t) \tag{2.5}$$

$$\lim_{y\to(-1)^m\infty} w_m\left(x,y,t\right) \to 0 \qquad \lim_{y\to(-1)^m\infty} \varphi_m\left(x,y,t\right) \to 0 \tag{2.6}$$

It is known that, when the coordinate axis $oz \| \bar{p}$ is parallel to the axes of the polarizations of all the composite crystals, in any plane ox_1x_2 (slice $x_3 = const$), quasi-static equations of electro elasticity in each layer

$$c_{ijkm}\frac{\partial^2 u_k^{(n)}}{\partial x_i \partial x_m} + e_{ijm}\frac{\partial^2 \varphi_n}{\partial x_i \partial x_m} = \rho_n \frac{\partial^2 u_j^{(n)}}{\partial t^2}, \qquad e_{ijm}\frac{\partial^2 u_j^{(n)}}{\partial x_i \partial x_m} - \varepsilon_{im}\frac{\partial^2 \varphi_n}{\partial x_i \partial x_m} = 0 \tag{2.7}$$

allow separation of non-electroactive plane strain state $\left\{u_1\left(x_1,x_2,t\right); u_2\left(x_1,x_2,t\right); 0; 0\right\}$ from electroactive anti-plane deformed state $\left\{0;0;u_3\left(x_1,x_2,t\right); \varphi\left(x_1,x_2,t\right)\right\}$ [18].

This will make it possible to separately investigate the effect of surface roughness on the propagation of the wave signal of flat deformation and the wave signal of electroactive anti-flat deformation.

In this case, the connection of heterogeneity environments in the formed composite is of geometric nature, and isdetermined by the functions of surface irregularities $\Sigma_m = \{y = h_m(x)\};\ m = 1;2.$ featuring in the boundary conditions (2.2) and (2.5).

Model-2. Two elastic deformable half-space made of materials characterized by associated physical fields (electro-elasticity, magneto-elasticity, thermos-elasticity, etc.) are connected by a method of thermal crushing, so that depressions of one of the rough surfaces were filledby protrusionsof the other.

Then at the junction of the connection of the half-spaces, a thin transversely-inhomogeneous layer with varying physical and mechanical characteristics of the material $\gamma^{(3)}(y) \triangleq \{c^{(3)}_{ijkn}(y); \rho^{(3)}(y); e^{(3)}_{mij}(y); \varepsilon^{(3)}_{nk}(y)\}$ is formed. Naturally, the virtual surfaces of the formed layer can be represented by lines (surfaces) of maximal depressions of the corresponding roughness. Then, allocating virtual layer thickness $H = (R_2 + R_1)/2$, where $R_m = |\max h_m(x) - \min h_m(x)|$ are the maximum value of the depressions of the rough edges, respectively, for functions $\gamma(y)$,the characteristic requirement will be the condition of equality of physicomechanical constant in planes $y = \pm(H/2)$ with $\gamma(-(H/2)) = \gamma^{(1)} \triangleq \{c^{(1)}_{ijkn}; \rho^{(1)}; e^{(1)}_{mij}; \varepsilon^{(1)}_{nk}\}$ and $\gamma(+(H/2)) =$, $= \gamma^{(2)} \triangleq \{c^{(2)}_{ijkn}; \rho^{(2)}; e^{(2)}_{mij}; \varepsilon^{(2)}_{nk}\}$ respectively. The heterogeneity of the material can be presented by any integrable function $\gamma^{(3)} = f(y, R_m, \gamma^{(m)});\ m = 1;2.$ with the following conditions on the surfaces of the virtual layer: $f(-(R_1 + R_2)/4) = \gamma_1$; $f((R_1 + R_2)/4) = \gamma_2$. As a result of diffusion of the deformed middle surface $y = (h_1(x) + h_2(x))/2$ of the formed non-homogeneous layer, it is natural to impose a new condition $f((h_1(x) + h_2(x))/2) = (\gamma_1 + \gamma_2)/2$.

For the studies of dynamic processes in $6mm$ piezoelectric hexagonal symmetry class with this model, with the indices of $n = 1;2$, the equations in (2.7) will be solved in homogeneous half-spaces $\Omega^*_1 = \{|x| < \infty, -\infty < y \le -(R_1 + R_2)/4, |z| < \infty\}$ and $\Omega^*_2 = \{|x| < \infty, (R_1 + R_2)/\quad 4 \le y < \infty, |z| < \infty\}$. In virtual dedicated thin layer $\Omega^*_3 = \{|x| < \infty, -(R_1 + R_2)/\ 4 \le y \le (R_1 + R_2)/4, |z| < \infty\}$ equations of electro-elasticity with variable coefficients will be solved:

$$c_{ijkm}(x_2)\frac{\partial^2 u^{(n)}_k}{\partial x_i \partial x_m} + e_{ijm}(x_2)\frac{\partial^2 \varphi_n}{\partial x_i \partial x_m} = \rho_n(x_2)\frac{\partial^2 u^{(n)}_j}{\partial t^2}$$

$$e_{ijm}(x_2)\frac{\partial^2 u^{(n)}_j}{\partial x_i \partial x_m} - \varepsilon_{im}(x_2)\frac{\partial^2 \varphi_n}{\partial x_i \partial x_m} = 0 \qquad (2.8)$$

Obviously, the study of the solutions to these equations become much more complicated, since variables coefficients lead to a nonlinear dependence of the amplitude and phase functions of the propagating wave signal [19], [20]. Butthe boundary conditions in (2.2) and (2.3) are given get much simpler. The change of the surfaces normal $\vec{n}^{(m)}\left(x\right)=\left\{n_j^{(m)}\left(x,h_m(x)\right)\right\}$ in surfaces $y=h_m(x)$, is replaced by the unit normal ofsmooth surfaces

$$n_j^{(m)}=\left\{0;\pm 1;0\right\}\ y=\pm(R_1+R_2)/4=const\ .$$

$$\sigma_{i2}^{(m)}(x,(-1)^m\cdot(R_1+R_2)/4,t)-\sigma_{i2}^{(3)}(x,(-1)^m\cdot(R_1+R_2)/4,t)=0\,, \qquad (2.2^*)$$

$$D_2^{(m)}(x,(-1)^m\cdot(R_1+R_2)/4,t)-D_2^{(3)}(x,(-1)^m\cdot(R_1+R_2)/4,t)=0\,, \qquad (2.3^*)$$

$$w_i^{(m)}(x,(-1)^m\cdot(R_1+R_2)/4,t)=w_i^{(3)}(x,(-1)^m\cdot(R_1+R_2)/4,t)\,, \qquad (2.4^*)$$

$$\varphi_m(x,(-1)^m\cdot(R_1+R_2)/4,t)=\varphi_3(x,(-1)^m\cdot(R_1+R_2)/4,t)\,, \qquad (2.5^*)$$

In this case the connection of environments, heterogeneity in the formed composite have a physical nature, and they are determined by the characteristics of the functions of surface irregularities $R_m=\left|\max h_m(x)-\min h_m(x)\right|$, which appear in the boundary conditions (2.2*) and (2.3*). Through the functions of the physicomechanical characteristics of the environment $\left\{c_{ijkn}^{(3)}(y);\rho^{(3)}(y);e_{mij}^{(3)}(y);\varepsilon_{nk}^{(3)}(y)\right\}$, heterogeneity appears also in the non-homogeneous equations piezoelectric (2.8) .

3. The mathematical modelling of boundary value problems. In the studies of wave processes in formed composites, for the analytical analysis of the process and the results, a hypothesis (MELS) of the nature of the distributions of characteristic physicomechanical units, or the thermodynamic parameters of the formed heterogeneous material, with the use of surface-exponential functions (SEF) is proposed. This hypothesis must ensure the existence of the characteristic parameters describing physical and mechanical fields, both in the equations, and in the boundary conditions of the problem.

Let's assume that a shear electro-elastic wave signal is distributed across the three-layer composite consisting of a lone polarized pasted piezoelectric glue of two thick layers of piezoelectric crystal of $6mm$ class hexagonal symmetry (model-1). Then, the plane strain state $\left\{u_1\left(x_1,x_2,t\right);u_2\left(x_1,x_2,t\right);0;0\right\}$ in the composite is not be inducting, and electro-active antiplane strain state $\left\{0;0;u_3\left(x_1,x_2,t\right);\varphi\left(x_1,x_2,t\right)\right\}$ is described by quasi-static solutions

$$w_n(x,y,t)=W_{0n}\exp((-1)^n\alpha_n ky)\cdot e^{i(kx-\omega t)}\,,$$

$$\varphi_n(x,y,t)=\left\{\Phi_{0n}\exp((-1)^n ky+\frac{e_n}{\varepsilon_n}W_{0n}\exp((-1)^n\alpha_n ky)\right\}\cdot e^{i(kx-\omega t)} \qquad (3.1)$$

where $G_n=c_{44}^{(n)}$ - elastic moduli shifts, ρ_n - density, $e_n=e_{15}^{(n)}$-piezoelectric modules, $\varepsilon_n=\varepsilon_{11}^{(n)}$ - relative coefficients of dielectric constant of materials in the respective layers $n=1,2,3$ of the composite, $\alpha_n^2=1-\left(\omega^2\rho_n^2\right)/\left(k^2\tilde{G}_n^2\right)$ -wave characteristic values; $\tilde{G}_n=G_n\left(1+e_n^2/(\varepsilon_n G_n)\right)$ -given shear stiffness of piezoelectric, ω- the frequency of a waveform, $k=\left(2\pi\right)/\lambda$ -wave number and λ-wavelength. In case of respective numbering $-n=1;2$; in the case of homogeneous half-spaces, amplitudes W_{0n} and Φ_{0n} are constants.

In the adhesive layer of variable thickness solutions are represented both in symmetric and anti-symmetric forms:

$$w_3(x,y,t) = \left\{ A_{31}\sin(\alpha_3 ky) + A_{32}\cos(\alpha_3 ky) \right\} \cdot e^{i(kx-\omega t)}$$

$$\varphi_3(x,y,t) = \left\{ B_{31}\sin(ky) + B_{32}\cos(ky) + \frac{e_3}{\varepsilon_3}\left[A_{31}\sin(\alpha_3 ky) + A_{32}\cos(\alpha_3 ky) \right] \right\} \cdot e^{i(kx-\omega t)} \quad (3.2)$$

To simplify the boundary conditions (2.2)-(2.5), we introduce hypotheses about distributions of the elastic displacements $w_3(x,y,t)$ and the potential of the electric field $\varphi_3(x,y,t)$ across the thickness of a thin layer of adhesive,

$$w_3(x,y,t) = \frac{e^{f(x)}-1}{e-1}\left[w_2(x,h_2(x),t) - w_1(x,h_1(x),t) \right] + w_1(x,h_1(x),t)$$

$$\varphi_3(x,y,t) = \frac{e^{f(x)}-1}{e-1}\left[\varphi_2(x,h_2(x),t) - \varphi_1(x,h_1(x),t) \right] + \varphi_1(x,h_1(x),t), \quad (3.3)$$

where surface-exponential function (SEF)

$$f(x,y) = \left[\exp\left[\frac{\left[y - h_1(x) \right]}{\left[h_2(x) - h_1(x) \right]} \right] - 1 \right] \cdot (e-1)^{-1} \quad (3.4)$$

is the characteristic function for non-smooth surfaces $y = h_m(x)$ and is described the changes of the requiredunknown valuesin the adhesive layer thickness. The introduced hypotheses of type (3.3) provide for any kind of conjugation related to physicomechanical fields in magneto- (electro-, thermo-) elastic layered systems (Magneto-Elastic Layered System ***(MELS) hypothesis***). They also allow to split boundary conditions (2.2)-(2.5) into two groups.

The conjugacy conditions of mechanical stress and electric induction (2.2) and (2.3)

$$h_1'(x)\left(\sigma_{31}^{(1)}(x,h_1(x),t) - \sigma_{31}^{(3)}(x,h_1(x),t) \right) + \left(\sigma_{32}^{(1)}(x,h_1(x),t) - \sigma_{32}^{(3)}(x,h_1(x),t) \right) = 0$$

$$h_2'(x)\left(\sigma_{31}^{(2)}(x,h_2(x),t) - \sigma_{31}^{(3)}(x,h_2(x),t) \right) + \left(\sigma_{32}^{(2)}(x,h_2(x),t) - \sigma_{32}^{(3)}(x,h_2(x),t) \right) = 0 \quad (3.5)$$

$$h_1'(x)\left(D_1^{(1)}(x,h_1(x),t) - D_1^{(3)}(x,h_1(x),t) \right) + \left(D_2^{(1)}(x,h_1(x),t) - D_2^{(3)}(x,h_1(x),t) \right) = 0$$

$$h_2'(x)\left(D_1^{(2)}(x,h_2(x),t) - D_1^{(3)}(x,h_2(x),t) \right) + \left(D_2^{(2)}(x,h_2(x),t) - D_2^{(3)}(x,h_2(x),t) \right) = 0 \quad (3.6)$$

are logged in the form of four algebraic equations with respect to four amplitude constants: $\left\{ A_j, B_j \right\} \in \left\{ W_{01}; W_{02}; \Phi_{01}; \Phi_{02} \right\}$

$$\hat{B}_{ij}\left[\{\gamma_3\}, h_m(x), h_m'(x), \alpha_n(\omega/k), \right] \times A_j = 0$$

From the condition of existence of nontrivial solutionswe obtain the transcendental dispersion equation, which determines the frequency response of the process

$$\det\left\| \hat{B}_{ij}\left[\{\gamma_3\}, h_m(x), n_j^{(m)}(x), \alpha_n(\omega/k), \right] \right\|_{4\times 4} = 0 \quad (3.7)$$

Another group of boundary conditions(2.4) and (2.5), aboutcontingency elastic displacements and electric potential with respect to four amplitude constants $A_j^* \in \left\{ A_{31}; A_{32}; B_{31}; B_{32} \right\}$, generates a new inhomogeneous system of four algebraic equations:

$$\hat{B}_{ij}^*\left[\{\gamma_n\}, h_m(x), h_m'(x), \alpha_n(\omega/k), \right] \times A_j^* = b_i\left[A_r, h_m(x), h_m'(x), \alpha_n(\omega/k), \right] \quad (3.8)$$

Hence obtained frequency-amplitudes descriptions for the identification of new phenomena at the joints between the layers of the composite are due to the roughness of the surfaces. The presence of roughness on the surfaces of the layers transforms the coefficients $\hat{B}_{ij}\left[\{\gamma_3\}, h_m(x), h'_m(x), \alpha_n(\omega/k),\right]$ and $\hat{B}^*_{ij}\left[\{\gamma_3\}, h_m(x), h'_m(x), \alpha_n(\omega/k),\right]$, as well as free terms $b_i\left[A_r, h_m(x), h'_m(x), \alpha_n(\omega/k),\right]$ in algebraic equations (3.7*) and (3.8) into complex variables. To examine the effect of roughness on dispersion, or the possibility of occurrence of resonance, the wave signal must be represented by a complex wave number and frequency fluctuationsc $F_n(x,y,t) \triangleq A_n(x,y)\cdot \exp i\{(k_1+ik_2)\cdot x-(\omega_1+i\omega_2)\cdot t\}$.

Then determinants from (3.7*) and (3.8) are real and imaginary parts of the wave number and frequency, with their signs will characterize the influence of roughness on the dispersion, dissipation and resonance in the wave signal propagation.

It was shown above that in the three-layer composite consisting of interconnected thermal breakdown, lone polarized, two thick piezoelectricity layers of hexagonal symmetry class $6mm$, a transversely inhomogeneous layer (model-2) is formed along the surfaces of the border connection:

$$\Omega_3^* = \{|x|<\infty, -(R_1+R_2)/4 \le y \le (R_1+R_2)/4, |z|<\infty\}$$

During the propagation of shear electro-elastic wave signal, in the composite, similar to the case in model-1, plane strain condition is not induced $\{u_1(x_1,x_2,t); u_2(x_1,x_2,t); 0; 0\}$, and the electroactiveantiplane strain state $\{0;0;u_3(x_1,x_2,t);\varphi(x_1,x_2,t)\}$ in thick layers is again describedby the quasi-static solutions in (3.1). Unlike the previous case, virtual dedicated heterogeneous in thickness layer Ω_3^*, the frequency-amplitude nature of the solutions will obviously depend on the initial roughness of surfaces through the heterogeneity of the material layer. Given the subtlety of the virtuallyformed inhomogeneous layer, in case of model-2, MELS hypotheses are introducedin relation to distributions of the physicomechanical diffusion constantsof the created environment $\gamma^{(3)}(y) \triangleq \{c^{(3)}_{ijkn}(y); \rho^{(3)}(y); e^{(3)}_{mij}(y); \varepsilon^{(3)}_{nk}(x)\}$, with the selected virtual surfaces of $y=\pm(R_1+R_2)/4 = const$: - $n_j^{(\pm)} = \{0;\pm 1;0\}$

$$\gamma^{(3)}(x,y) = \frac{e^{f_\gamma(x,y)}-1}{e-1}\left(\gamma^{(2)}-\gamma^{(1)}\right)+\gamma^{(1)} \qquad (3.9)$$

$$f_\gamma(x,y) = a(\gamma_m, R_m, h_m(x))\cdot y^2 + b(\gamma_m, R_m, h_m(x))\cdot y + c(\gamma_m, R_m, h_m(x)) \qquad (3.10)$$

The characteristic function describing the change of the thermodynamic constants for the layer thickness, $\gamma^{(1)} \triangleq \{c^{(1)}_{ijkn}; \rho^{(1)}; e^{(1)}_{mij}; \varepsilon^{(1)}_{nk}\}$ and $\gamma^{(2)} \triangleq \{c^{(2)}_{ijkn}; \rho^{(2)}; e^{(2)}_{mij}; \varepsilon^{(2)}_{nk}\}$ are thermodynamic constants of the materials of the respective half-spaces, $R_m = |\max h_m(x) - \min h_m(x)|$ -the depth of the depressions corresponding to the surface roughness.The coefficients of the functions $f_\gamma(y)$ are selected from the attainment of material values of the thermodynamic parameters of the virtually created environment in the border layer

$$\gamma^{(3)}\left(-\frac{R_1+R_2}{4}\right)=\gamma^{(1)}, \quad \gamma^{(3)}\left(\frac{R_1+R_2}{4}\right)=\gamma^{(2)}, \quad \gamma^{(3)}\left(\frac{h_1(x)+h_2(x)}{2}\right)=\frac{\gamma^{(2)}+\gamma^{(1)}}{2} \qquad (3.11)$$

By introducing hypothesis MELS (3.9) and an exponential function of surface SEF (3.10) we obtain the boundary value problem of electro-elasticity from the following equations:

$$\tilde{G}_n\left[\frac{\partial^2 w_n}{\partial x^2}+\frac{\partial^2 w_n}{\partial y^2}\right]=\rho_n\frac{\partial^2 w_n}{\partial t^2}, \qquad \frac{\partial^2 \varphi_n}{\partial x^2}+\frac{\partial^2 \varphi_n}{\partial y^2}=\frac{e_n}{\varepsilon_n}\left[\frac{\partial^2 w_n}{\partial x^2}+\frac{\partial^2 w_n}{\partial y^2}\right] \qquad (3.12)$$

where $n=1,\ 2.$ in the half-spaces Ω_1^* and Ω_2^* accordingly with the solutions of (3.1), and equations:

$$\begin{gathered}
G_3(x,y)\left[\frac{\partial^2 w_3}{\partial x^2}+\frac{\partial^2 w_3}{\partial y^2}\right]+e_3(x,y)\left[\frac{\partial^2 \varphi_3}{\partial x^2}+\frac{\partial^2 \varphi_3}{\partial y^2}\right] \\
\frac{\partial G_3}{\partial x}\frac{\partial w_3}{\partial x}+\frac{\partial G_3}{\partial y}\frac{\partial w_3}{\partial y}+\frac{\partial e_3}{\partial x}\frac{\partial \varphi_3}{\partial x}+\frac{\partial e_3}{\partial y}\frac{\partial \varphi_3}{\partial y}=\rho_3(x,y)\frac{\partial^2 w_3}{\partial t^2} \\
e_3(x,y)\left[\frac{\partial^2 w_3}{\partial x^2}+\frac{\partial^2 w_3}{\partial y^2}\right]-\varepsilon_3(x,y)\left[\frac{\partial^2 \varphi_3}{\partial x^2}+\frac{\partial^2 \varphi_3}{\partial y^2}\right]+ \\
\frac{\partial e_3}{\partial x}\frac{\partial w_3}{\partial x}+\frac{\partial e_3}{\partial y}\frac{\partial w_3}{\partial y}-\frac{\partial \varepsilon_3}{\partial x}\frac{\partial \varphi_3}{\partial x}-\frac{\partial \varepsilon_3}{\partial y}\frac{\partial \varphi_3}{\partial y}=0
\end{gathered} \qquad (3.13)$$

in virtual dedicated inhomogeneous layer Ω_3^* .

Conditions for the existence of solutions of $F(x,y,t)\triangleq A(x,y)\cdot\exp i\{\vartheta(x,y)-\omega t\}$ equations (3.13), and their solution at the selection of functions for material heterogeneity $\gamma^{(3)}(x,y)$ with different combinations of boundary terms, have been discussed in articles [19-21]. The given boundary conditions (2.2*)-(2.5*) on the surfaces of the virtual layer $y=\pm(R_1+R_2)/4=const$ and the terms of the decay at infinity (2.6) together with the obtained equations (3.12) and (3.13) form a boundary value problem in model-2 connection piezoelectricity half-spaces with rough surfaces.The task here is to study complex transcendental equations. By introducing the wave number $k=k_1+ik_2$, and frequency fluctuations $\omega=\omega_1+i\omega_2$ in aggregate form into the wave signal, we will identify the influence of surface roughness on dispersion, dissipation and resonance in the wave process.

4. About hypotheses MELS. We have already mentioned that the hypotheses (3.3) and (3.9) provide conjugation of elastic displacement and the potential of electric fields on rough discontinuity surfaces (2.4) and (2.5) in model-1, or the equality of thermodynamic constants on the surfaces of the virtual layer in model-2 .It is obvious that these hypotheses are also suitable in problems of composites with materials associated with other physicomechanical fields (magneto-elasticity, thermos-elasticity, etc.) Another characteristic property of the introduced hypotheses is hidden in the choice of surface-exponential functions SEF (3.4) and (3.10), distribution across the thickness ofrequired physicomechanical fields (3.3), or the change of the thermodynamic constants of virtually created environment (3.9). Upon selection of a simpler distribution functions in thickness, such as

$$f(x,y)=\left[\frac{y-h_1(x)}{h_2(x)-h_1(x)}\right]^m, \quad m\in\{1;2;...\} \tag{4.1}$$

Although it provides conjugation of certain selected components ofphysicaomechanical fields, depending on the choice of measure $m\in\{1;2;...\}$,there’s always the risk of losing of some even or odd characteristic values in the study material. In this respect, the introduction of SEF – functions

$$f(x,y)=\frac{1}{e-1}\left\{\exp\left[\frac{y-h_1(x)}{h_2(x)-h_1(x)}\right]-1\right\} \text{ or } f_\gamma(y)=\frac{1}{e-1}\left\{\exp\left[\frac{y+(R_1+R_2)/4}{(R_2+R_1)/2}\right]-1\right\} \tag{4.2}$$

is acceptable not only because they provide the conjugation of the requiredphysicaomechanical fields or thermodynamic constants on rough surfaces, but also they introduce existing geometric surface roughness heterogeneity $y=h_m(x)$ into the descriptions of physicomechanical fields or thermodynamic constants ofthe inner thin layer.

5. Conclusions. When homogeneous layers get into contact with rough surfaces, geometrically or physically inhomogeneous layer is formed in the borderline area of the contact. Two model compounds of the rough surfaces in the form of piezoelectric sandwich of the composite are selected. Inputted superficially exponential functions SEF constructed hypotheses about distributions of physical, mechanical and thermodynamic fields of the permanent magneto- (electro-, thermo-, etc.) elastic layered systems-MELS, formed in an inhomogeneous layer connection. The introduction of hypotheses allows to model a mathematical boundary-value problem for different materials of the composite layers associated with different physicomechanical fields. A comparative numerical analysis of results with different boundary problem modelling will be given by the author in the following articles.

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The Boundary Problem Molelling of Rough Surfaces Continuous Media with Coupled Physicomechanical Fields

The presented hypotheses of magnetic- (electro-/thermo-) elastic layered systems (hypotheses-MELS) are addressed to the modeling of boundary problem of contact of rough surfaces of continuous media with associated physical and mechanical fields. In various models of joints of thick piezoelectric layers combinations is allocated exponential behaviour of physicomechanical fields or thermodynamic constants are given. The effect of surfaces roughness on the equations and the thermodynamic relations is achieved through the choice of surface-exponential functions (SEF).

ՀՀ ԳԱԱ թղթակից անդամ Ա. Ս. Ավետիսյան

Ֆիզիկամեխանիկական կապակցված դաշտերով հոծ միջավայրերի անհարթություններով եզրերի ամրացման եզրային խնդրի մոդելավորումը

Ներմուծելով մագնիսա (էլեկտրա, թերմո) առաձգական շերտավոր համակարգերի հիփոթեզներ (hiphotesis-MELS)՝ իրականացվում է կապակցված ֆիզիկամեխանիկական դաշտերով հոծ միջավայրերի անհարթ մակերնույթների ամրակցման եզրային խնդրի մոդելավորում: Պիեզոդիէլեկտրիկ շերտերի երկու տարբեր ամրակցումների դեպքերում առանձնացվում է ամրակցման մերձմակերևութային՝ ֆիզիկորեն կամ երկրաչափորեն անհամասեռ շերտ, որտեղ մակերևութային-աստիճանային ֆունկցիաների (SEF) միջոցով տրվում են ֆիզիկամեխանիկական դաշտերի կամ թերմոդինամիկական հաստատունների բաշխման վարքերը:

Այդ ֆունկցիաների ընտրությամբ իրականացվում է մակերևութային անհարթությունների ազդեցությունը եզրային խնդրի հավասարումներում և նյութական առնչություններում:

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Моделирование граничной задачи контакта шероховатых поверхностей сплошных сред со связанными физико-механическими полями

Вводом гипотез магнито (электро, термо) упругих слоистых систем (гипотезы – MELS) проводится моделирование граничной задачи контакта шероховатых поверхностей сплошных сред со связанными полями. При разных моделях соединений толстых пьезодиэлектрических слоев выделяется приповерхностный геометрически или физически неоднородный слой, в котором задается поверхностно-экспоненциальное поведение физико-механических полей или термодинамических постоянных. Выбором поверхностно-экспоненциальной функции (SEF) обеспечивается влияние поверхностной шероховатости в уравнениях и в термодинамических соотношениях задачи.

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