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Rigid Body Points Approximating Concentric Spheres in Alternating Sets of its Given Positions

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Introduction. In [1] we have first studied the special points of a body which in N positions during a given co-planar motion remain as close as possible to a circle, using as the measure of closeness the sum of squared distances of such points from the associated approximating circles. In the follow up paper [2] the results of this study have been extended to the points approximating a sphere in a given spatial motion.

In this paper, the concepts of [1] and [2] are generalized for the case of multiple approximating spheres with a common center corresponding to alternating sets of rigid body positions. Earlier, approximations (fittings) of 3D data point sets by concentric spheres were studied in computer metrology [3, 4] and more intensively in physiotherapy applications, in connection with the determination of the center of relative rotation of two adjacent segments constituting a human joint [5-7].

The present paper considers a new and more complex problem. We seek to determine the points of a moving body which in its m given sets of finitely separated positions remain as close as possible to corresponding m concentric spheres. By analogy with [2], as a tool to find these points we use the least square approximations with an algebraic error function defined below.

Least square approximations of spatial point position sets by concentric spheres. A rigid body e undergoes spatial motion with respect to a fixed body E. Coordinate systems oxyz and OXYZ are rigidly attached to e and E respectively. We consider the following problem: given a *point B* (x_B, y_B, z_B) in e, determine a set of spheres λ_j^E $(j = 1, 2, \dots, m)$ in E with a common center A radii R_j $(j= 1, 2, \dots, m)$, so that each j-th sphere of this set is as close as possible to the assigned j-th set of points B_{ji} $(i= 1, 2, \dots, N_j)$. In other words, the sought-for spheres should minimize in all $N = \sum_{j=1}^{m} N_j$ given positions of e geometric (radial) deviations Δ_{ji} of points B_{ji} from these spheres*:*

$$\Delta_{ji} = \left|\overline{AB_{ji}}\right| - R_j = \left(R_{B_{ji}}^2 + R_A^2 - 2\overline{R}_{B_{ji}}\overline{R}_A\right)^{\frac{1}{2}} - R_j\,, \qquad (1)$$

where $\overline{R}_{B_{ji}}$ and $\overline{R}_A$ are the position vectors of points B_{ji} and A from the origin 0.

The objective function estimating the closeness of points B_{ji} to the approximating sphere λ_j may have different forms depending on the selected criteria of closeness (approximation measure). As mentioned above, here we use the least square objective which requires to minimize the sum of squared radial deviations (1) of points B_{ji} from λ_j^E. However, there is no closed form solution for the approximating sphere based on this objective since the sphere center coordinates X_A, Y_A, Z_A are in the radicand of (1), and a lengthy iterative search routine is required to determine them. In order to avoid these computational difficulties we use another error function proposed in [2] for the single sphere case:

$$\Delta q_{ji} = \left|\overline{AB_{ji}}\right|^2 - R_j^2 = \Delta_{ji}\left(\overline{AB_{ji}} + R_j\right). \qquad (2)$$

Clearly, $\Delta q_{ji} = 0$ if only point B_{ji} lies on the sphere λ_j^E, also, Δq_{ji} is small if the point lies near the sphere. Now, since $\left(\overline{AB_{ji}}\right)^2 = \left(R_j + \Delta_{ji}\right)^2 =$ $= R_j^2 + R_j\Delta_{ji} + \Delta_{ji}^2$, it follows from (2) that if points B_{ji} are rather close to λ_j^E, and Δ_{ji}^2 can be neglected, then $\Delta q_{ji} \approx 2R_j\Delta_{ji}$, i.e. Δq_{ji} is proportional to Δ_{ji} and can substitute it in the further minimization procedures. To determine the approximating concentric spheres λ_j^E (j= 1, 2, …, m) we transform (2) to the following linear function:

$$\Delta q_{ji} = -2\left(X_{B_{ji}}X_A + Y_{B_{ji}}Y_A + Z_{B_{ji}}Z_A + H_j - \frac{1}{2}R_{B_{ji}}^2\right), \qquad (3)$$

where H_j is a constant depending only on the sphere parameters:

$$H_j = \frac{1}{2}\left(R_j^2 - X_A^2 - Y_A^2 - Z_A^2\right), \qquad (4)$$

Now we form the objective function as the sum of squared algebraic deviations determined for m subsets of the given positions e_{ji} *(j=1, 2, …, m; i=1, 2, …,* N_j*):*

$$S = \sum_{j=1}^{m}\sum_{i=1}^{N_j}\Delta_{q_{ji}}^2\ . \qquad (5)$$

For any point B (x_B, y_B, z_B) given in e the concentric spheres λ_j^E (j= 1, 2, …, m) approximating the corresponding sets of positions B_{ji} of B in E should be determined from the necessary conditions for a minimum of (5):

$$\frac{\partial S}{\partial X_A} = 0\,,\ \ \frac{\partial S}{\partial Y_A} = 0\,,\ \ \frac{\partial S}{\partial Z_A} = 0\,,\ \ \frac{\partial S}{\partial R_1} = 0\,,\ \ldots,\ \frac{\partial S}{\partial R_m} = 0\ . \qquad (6)$$

After substituting into (5) from (4) and using for brevity notations X_{ji}, Y_{ji}, Z_{ji}, R_{ji} for $X_{B_{ji}}, Y_{B_{ji}}, Z_{B_{ji}}$ and $R_{B_{ji}}$ respectively, conditions (6) can be reduced to the following system of (3+m) linear equations in X_A, Y_A, Z_A, H_j (j= 1, 2, …, m) presented below in a matrix form:

$$M^E P^E = F^E , \tag{7}$$

where

$$M^E = \begin{bmatrix} \sum_{j=1}^{m}\sum_{i=1}^{N_j} X_{ji}^2 & \sum_{j=1}^{m}\sum_{i=1}^{N_j} X_{ji}Y_{ji} & \sum_{j=1}^{m}\sum_{i=1}^{N_j} X_{ji}Z_{ji} & \sum_{i=1}^{N_1} X_{1i} & \dots & \sum_{i=1}^{N_1} X_{mi} \\ \sum_{j=1}^{m}\sum_{i=1}^{N_j} X_{ji}Y_{ji} & \sum_{j=1}^{m}\sum_{i=1}^{N_j} Y_{ji}^2 & \sum_{j=1}^{m}\sum_{i=1}^{N_j} Y_{ji}Z_{ji} & \sum_{i=1}^{N_1} Y_{1i} & \dots & \sum_{i=1}^{N_m} X_{mi} \\ \sum_{j=1}^{m}\sum_{i=1}^{N_j} X_{ji}Z_{ji} & \sum_{j=1}^{m}\sum_{i=1}^{N_j} Y_{ji}Z_{ji} & \sum_{j=1}^{m}\sum_{i=1}^{N_j} Z_{ji}^2 & \sum_{i=1}^{N_1} Z_{1i} & \dots & \sum_{i=1}^{N_m} Z_{mi} \\ \sum_{i=1}^{N_1} X_{1i} & \sum_{i=1}^{N_1} Y_{1i} & \sum_{i=1}^{N_1} Z_{1i} & N_1 & \dots & 0 \\ \sum_{i=1}^{N_2} X_{2i} & \sum_{i=1}^{N_2} Y_{2i} & \sum_{i=1}^{N_2} Z_{2i} & 0 & \dots & 0 \\ . & . & . & . & \dots & . \\ . & . & . & . & \dots & . \\ \sum_{i=1}^{N_m} Z_{1i} & \sum_{i=1}^{N_m} Y_{mi} & \sum_{i=1}^{N_m} Z_{mi} & 0 & \dots & N_m \end{bmatrix},$$

$$P^E = \left[X_A, Y_A, Z_A, H_1, \dots, H_m \right]^t ,$$

$$F^E = \left[\sum_{j=1}^{m}\sum_{j=1}^{N_j} X_{ji}R_{ji}^2, \sum_{j=1}^{m}\sum_{j=1}^{N_j} y_{ji}R_{ji}^2, \sum_{j=1}^{m}\sum_{j=1}^{N_j} Z_{ji}R_{ji}^2, \dots \sum_{i=1}^{N_j} R_{mi}^2 \right].$$

It is easy to see from (3-5) that conditions $\frac{\partial S}{\partial R_j} = 0$ in (7) are equivalent to $\frac{\partial S}{\partial H_j} = 0$.

To solve the system (8) we express coordinates X_{ji}, Y_{ji}, Z_{ji} of point B in E through its coordinates x_B, y_B, z_B in e by means of the following linear transformation :

$$\begin{bmatrix} X_{ji} \\ Y_{ji} \\ Z_{ji} \end{bmatrix} = \begin{bmatrix} X_{oji} \\ Y_{oji} \\ Z_{oji} \end{bmatrix} + T_{ji} \begin{bmatrix} x_B \\ y_B \\ z_B \end{bmatrix}, \tag{8}$$

where T_{ji} is a 3x3 rotation matrix which rotates the system xyz from a position with its axes initially parallel to the axes X, Y, Z to its i-th position of j-th given position set e_{ji}.

It follows then that if we have m sets of positions e_{ji}*(j=1, 2, …,m; i=1, 2, …,* N_j*)* and if we select an arbitrary point B in e, equation (7) uniquely determines m spheres λ_j^E $(j=1,2,\dots,m)$ with the common center $A(X_A, Y_A, Z_A)$ approximating the corresponding m position sets B_{ji} *(j=1, 2, …, m; i=1, 2, …, N_j)* of point B given by (8). For the further analysis, it is convenient to present the solution of (7) in the vector form:

$$\left(X_A, Y_A, Z_A, H_1, \dots, H_m\right) = \frac{1}{D}\left(D_X, D_Y, D_Z, D_{H_1}, D_{H_2}, \dots, D_{H_m}\right) \tag{9}$$

where $D_X, D_Y, D_Z, D_{H_1}, D_{H_2}, \dots, D_{H_n}$ are (3+m)-th order determinants defined by the expanded matrix of (7). The solution (9) exists and is uniquely determined, unless the coefficient matrix m of the system (7) is singular. This can happen if only all points Bji are coplanar [2].

After determining the coordinates of the common center A of spheres λ_j and constants H_j by (10), we find the radii R_j of λ_j from (4):

$$R_j = \left(X_A^2 + Y_A^2 + Z_A^2 + 2H_j\right)^{\frac{1}{2}},\ j=1,2,\dots,m\ .$$

Substituting (4) for H_j into the last m equations of (7), we obtain another expression for R_j:

$$R_j = \frac{1}{N_j}\sum_{i=1}^{N_j}\left[\left(X_{ji} - X_A\right)^2 + \left(Y_{ji} - Y_A\right)^2 + \left(Z_{ji} - Z_A\right)^2\right]^{\frac{1}{2}},\ j=1,2,\dots,m\ . \tag{10}$$

It follows from (10) that the radius of each j-th approximating sphere λ_j is the root-mean-square of distances A B_{ji}(i=1, 2, …, N_j) between the common center A of all spheres λ_j and N_j positions B_{ji} (i=1, 2, …, N_j) of B in the j-th set of prescribed positions e_{ji} (i=1, 2, …, N_j).

Correspondence between points of e and E. Determinants in the right side of (9) are functions of the coordinates x_B, y_B, z_B of B in e. It follows from (10) then that corresponding to any point B in e there is a unique fixed point in E which is the common center of spheres λ_j (j=1, 2, …, m) approximating in the last square sense given m alternating sets of point-positions B_{ji} (j=1, 2, …, m; i=1, 2, …, N_j).

If we invert the moving and fixed bodies so that e becomes the fixed body and E moves so as to maintain the same relative positions as in the original motion, we obtain the following expression for error functions $\Delta_{q_{ji}}$:

$$\Delta_{q_{ji}} = \left(\bar{r}_B - \bar{r}_A\right)^2 - R_j^2 - 2\left(x_B x_{A_{ji}} + y_B y_{A_{ji}} + z_B Z_{A_{ji}} + h_j - \frac{1}{2} r_{B_{ji}}^2\right), \tag{11}$$

where $\bar{r}_B\left(x_B, y_B, z_B\right)$ and $\bar{r}_{A_{ji}}\left(x_{A_{ji}}, y_{A_{ji}}, z_{A_{ji}}\right)$ are vectors from the origin o_{ji} in e to points B and A_{ji} respectively, while h_j are constants depending on the parameters of concentric spheres λ_j to be determined:

$$h_j = \frac{1}{2}\left(R_j^2 - r_B^2\right) = \frac{1}{2}\left(R_j^2 - x_B^2 - y_B^2 - z_B^2\right),\ j=1,2,\dots,m\ . \tag{12}$$

Assuming we have a point A in E, the corresponding m concentric spheres λ_j^e in e which approximate best m sets of inverted positions A_{ji} *(j=1, 2, …, m; i=1, 2, …, N_j)* of A should satisfy the necessary conditions for a minimum of (5):

$$\frac{\partial S}{\partial x_B}=0,\ \frac{\partial S}{\partial y_B}=0,\ \frac{\partial S}{\partial z_B}=0,\ \frac{\partial S}{\partial h_1}=0,\ldots,\frac{\partial S}{\partial h_m}=0\,. \tag{13}$$

Substituting into (13) from (5) and (11), we can transform conditions (13) into a system of (3+m) linear equations in x_B, y_B, z_B, h_1, …, h_m which we present in the matrix form assuming notations by analogy with (7):

$$M^e p^e = F^e\,. \tag{14}$$

Expressions of M^e, P^e and F^e in (14) are similar to those for M^E, P^E and F^E in (7) and can be easily written by a simple change of notations.

Coordinates $x_{A_{ji}}, y_{A_{ji}}, z_{A_{ji}}$ of inverted positions A_{ji} of point A in e can be determined by the following formula of linear transformation:

$$\left[x_{A_{ji}}, y_{A_{ji}}, z_{A_{ji}}\right]^t = T_{ji}^{-1}\left[X_A - X_{oji}, Y_A - Y_{0j}, Z_A - Z_{oji}\right]^t. \tag{15}$$

where T_{ji}^{-1} is 3x3 matrix inverse to T_{ji} in (8).

Equation (14) establishes a correspondence between points A of E and the centers B of concentric sphere sets $\left\{\lambda_j^e\right\}$ in e approximating m sets of inverted positions A_{ji} (j=1, 2, …, m; i=1, 2, …, N_j) defined by (15). The singular case of this correspondence for m=1 is studied in [2].

Points of e deviating least from concentric spheres. Now we proceed to the main issue in this study: which points of e will approximate best concentric spheres λ_j^E in alternating sets of given positions *e_{ji}(j=1, 2,…, m; i=1, 2,…, N_j).* The sum (5) is a function of (6+m) variables: X_A, Y_A, Z_A, x_B, y_B, z_B, R_j (j=1, 2, …, m). Therefore, for S to be a minimum, *the following conditions are necessary:*

$$\frac{\partial S}{\partial X_A}=0,\frac{\partial S}{\partial Y_A}=0,\frac{\partial S}{\partial Z_A}=0,\frac{\partial S}{\partial x_B}=0,\frac{\partial S}{\partial y_B}=0,\frac{\partial S}{\partial Z_B}=0,\frac{\partial S}{\partial R_j}=0\ \left(j=1,2,\ldots,m\right) \tag{16}$$

It is easy to see that equations (16) can be obtained by combining systems (6) and (13) discussed above. This means that any set of the sought-for (6+m) parameters for which S has a minimum should satisfy to equations (6) and (13).The foregoing leads to the following theorem which gives a geometric interpretation to the conditions (16).

Theorem. *In order for a moving point-fixed center pair (B, A) to cause the sum (5) to be a minimum it is necessary that:*

1) *A be a common center of spheres $\lambda_j^E\left(j=1,2,\ldots,m\right)$ approximating m sets of positions B_{ji} (j=1, 2, …, m; i= 1, 2, …, N_j) of point B which it occupies in given positions e_{ji} of e with respect to E,*
2) *B be the common center of spheres $\lambda_j\left(j=1,2,\ldots,m\right)$ approximating m sets of inverted positions A_{ji} (j=1, 2, …, m; i= 1, 2, …, N_j) of point A which it occupies in inverted positions E_{ji} of E with respect to e.*

To study the locus of points in e for which S has stationary values, we first present the 4-th, 5-th and 6-th equations of (16) in the following form:

$$\sum_{j=1}^{m}\sum_{i=1}^{N_j}\Delta q_{ji}\frac{\partial \Delta q_{ji}}{\partial x_B}=0\,,\ \sum_{j=1}^{m}\sum_{i=1}^{N_j}\Delta q_{ji}\frac{\partial \Delta q_{ji}}{\partial y_B}=0\,,\ \sum_{j=1}^{m}\sum_{i=1}^{N_j}\Delta q_{ji}\frac{\partial \Delta q_{ji}}{\partial z_B}=0\,,$$

then substitute in them relations $\frac{\partial \Delta_{q_{ij}}}{\partial x_B} = -2x_{A_{ij}}$, $\frac{\partial \Delta_{q_{ij}}}{\partial y_B} = -2y_{A_{ij}}$, $\frac{\partial \Delta_{q_{ij}}}{\partial z_B} = -2z_{A_{ij}}$ following from (11) and expression (3) for $\Delta_{q_{ij}}$, which yields the following 3 equations:

$$\sum_{j=1}^{m}\sum_{i=1}^{N_j}\left(X_{ji}X_A + Y_{ji}Y_A + Z_{ji}Z_A + H_j - \frac{1}{2}R_{ji}^2 \right)x_{A_j} = 0 ,$$

$$\sum_{j=1}^{m}\sum_{i=1}^{N_j}\left(X_{ji}X_A + Y_{ji}Y_A + Z_{ji}Z_A + H_j - \frac{1}{2}R_{ji}^2 \right)y_{A_j} = 0 , \qquad (17)$$

$$\sum_{j=1}^{m}\sum_{i=1}^{N_j}\left(X_{ji}X_A + Y_{ji}Y_A + Z_{ji}Z_A + H_j - \frac{1}{2}R_{ji}^2 \right)z_{A_j} = 0 .$$

Substituting in (17) expressions of $x_{A_{ji}}, y_{A_{ji}}, z_{A_{ji}}$ from (15) and relations $X_A=D_X/D$, $Y_A=D_Y/D$, $Z_A=D_Z/D$ from (10), after some transformations we can express equations (17) as

$$V_l = K_1^l D_X^2 + K_2^l D_Y^2 + K_3^l D_Z^2 + \sum_{j=1}^{m} K_{4j}^l D_X D_{H_j} + \sum_{j=1}^{m} K_{5j}^l D_y D_{H_j} + \sum_{j=1}^{m} K_{6j}^l D_Z D_{H_j} + K_7^l D_X D +$$

$$K_8^l D_X D K_9^l D_X D + \sum_{j=1}^{m} K_{10j} D_{H_j} D + K_{11}^l D^2 + K_{12}^l D_X D_Y + K_{13}^l D_X D_Z + K_{14}^l D_Y D_Z = 0 \left(l = x, y, z \right)$$

where coefficients $K_1^l \ldots K_{14}^l \left(l = x, y, z \right)$ are linear or zero order functions of x_B, y_B, z_B.

V_x, V_y, V_z in (18) are homogenous quadratic forms in $D, D_X, D_Y, D_Z, D_{H_j}$. The analysis carried out in [2] for the case of a single set of given positions ($m=1$) has shown that determinants D, D_X, D_Y, D_Z can be expanded into 6-th order and $D_{H_j}\left(j=1,2,\ldots,m \right)$ into 8-th order polynomials in x_B, y_B, z_B. Furthermore, it has been established that equations (18) studied for $m=1$ define 3 algebraic surfaces of 13-th order embedded in e. Points of intersection of these surfaces correspond to the stationary values of the objective function (5). It has been proved in [8] that the maximum number of real points which can generally satisfy the equations (18) is not more than 245. It is easy to be convinced that these results do not depend on the number m of position sets and remain valid for $m>1$, too. This indicates that, in general, it is likely to expect a large number of points in e which bring to a minimum of S. Among them, we should determine the points which approximate concentric spheres with a sufficient accuracy.

An interactive method for determining the local minimums of S. Algebraic deviations (distances) Δq_{ji} of point-position sets considered above from a sphere are bilinear functions of (6+m) parameters X_A, Y_A, Z_A, x_B, y_B, z_B, H_j *(j=1, 2, ..., m)* which can be represented as linear functions (5) and (12) by fixing a point *B* (x_B, y_B, z_B) in e and A point *A* (X_A, Y_A, Z_A) in *E* respectively. This property allows to avoid the solution of the nonlinear system (19) in determining local minimums of S and to use the method of successive linear iterations developed in [8] for locating the so called least square circle points of a moving plane which approximate a circle in a given planar motion. The basic idea of the method follows from a theorem formulated in [8] which is applicable to any motion approximation problem with an associated bilinear error function. We reformulate it below in terms of the problem under consideration.

Theorem 2. *If $S^{(1)}$ is the value of S for the approximating concentric sphere set $\{\lambda_j^E\}^{(1)}$ with the center $A^{(1)}$ determined by (9) for m sets of positions $B_{ji}^{(1)}$ $(j=1,2,\ldots,m;\ i=1,2,\ldots,N_j)$ of an arbitrary point $B^{(1)}$ of e and $S_0^{(1)}$ is the value of S for the approximating concentric sphere set $\{\lambda_j^e\}^{(1)}$ with the center $A^{(1)}$ determined by (14) for m sets of the inverted positions $A_{ji}^{(1)}$ $(j=1,2,\ldots,m;\ i=1,2,\ldots,N_j)$ of $A^{(1)}$, then $S_0^{(1)} \le S^{(1)}$.*

It is easy to be convinced that the proposition of the theorem is valid also for the next and further inversions of relative positions of systems e and E, i.e. the approximating sphere set $\{\lambda_j^E\}^{(2)}$ constructed for m sets of positions $B_{ji}^{(2)}(j=1,2,...,m; i=1,2,...,N_j)$ of $B^{(2)}$ will yield a value $S^{(2)}$ smaller then $S^{(1)}$, etc. Continuing this process of successive kinematic inversions, we get a series of linear multiple sphere approximation problems with a decreasing sequence $S^{(1)}, S_0^{(1)}, S^{(2)}, S_0^{(2)}....$ of the objective function values which converges at one of local minimums of S. The iteration process is terminated when the prescribed accuracy of solution is achieved. The described method was tested by the numerical examples of designing a 5 (SPS) manipulator with adjustable link-lengths for the approximate generation of 2 given sets of the rigid body positions [9].

Conclusion. We have presented a study of special points of a rigid body which in m alternating sets of given positions deviate least in a least square sense from concentric spheres. The results of this paper can be considered as a generalization of the theory of so called least square sphere points developed in [2] for the case of a single set of rigid body positions. Similar to the case of a single position set, it is established that the locus of the sought-for points lies at the intersection of three 13^{th} order algebraic surfaces. An efficient iteration method of determining these points is proposed which can be readily applied to the synthesis of reconfigurable platform-type manipulators with spherical joints and adjustable link-lengths designed for the approximate generation of given multiphase motions or multiple point-paths. It is anticipated to expand this study by including other geometrical entities corresponding to the joint constraints in spatial mechanisms, such as parallel planes, coaxial cones and cylinders.

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Rigid Body Points Approximating Concentric Spheres in Alternating Sets of its Given Positions

The problem of determining the points of a body which in alternating sets of its given positions deviate least in the least square sense from concentric spheres is considered. The sought-for approximation is one which minimizes the sum of squared algebraic deviations of these points from the concentric spheres approximating their paths in each of the given sets of positions. The points of interest lie at the intersection of three 13^{th} order surfaces corresponding to the stationary conditions of the least square objective function. The theory and methods developed here can be applied to the synthesis of spatial adjustable mechanisms for the approximate generation of multi-phase motions or multiple point-paths.

Ակադեմիկոս Յու.Լ. Սարգսյան

Պինդ մարմնի տված դիրքերի իրարահաջորդ բազմություններում համակենտրոն գնդեր մոտարկող կետերի մասին

Դիտարկվում է պինդ մարմնի այնպիսի կետերի որոշման խնդիրը, որոնք նրա նախանշված դիրքերի իրարահաջորդ բազմություններում ամենաքիչն են շեղվում համակենտրոն սֆերաներից՝ նվազագույն քառակուսիների իմաստով: Հաշվարկվող մոտարկումը նվազարկում է որոնելի կետերի համակենտրոն սֆերաներից հանրահաշվական շեղումների քառակուսիների գումարը տված դիրքերի համապատասխան բազմություններում: Ուսումնասիրության առարկա կետերը որոշվում են միջին քառակուսային շեղման նպատակային ֆունկցիայի ստացիոնարության պայմաններն արտապատկերող 13-րդ կարգի երեք հանրահաշվական մակերևույթների փոխհատումով: Հոդվածում ներկայացված տեսությունն ու մեթոդները կարող են անմիջականորեն կիրառվել կարգավորվող տարածական մանիպուլյացիոն մեխանիզմների սինթեզում, որոնք նախատեսված են տված բազմափուլ շարժումների կամ բազմակի հետագծերի մոտավոր վերարտադրության համար:

Академик Ю. Л. Саркисян

Точки твёрдого тела, аппроксимирующие концентрические сферы в чередующихся множествах его заданных положений

Рассматривается задача определения точек твердого тела, которые в чередующихся множествах его положений наименее уклоняются от концентрических сфер в смысле наименьших квадратов. Искомое приближение минимизирует сумму наименьших квадратов алгебраических отклонений (расстояний) указанных

точек от концентрических сфер в соответствующих множествах заданных положений. Интересующие нас точки лежат на пересечении трёх алгебраических поверхностей тринадцатого порядка, отображающих условия стационарности целевой функции среднеквадратического отклонения. Теория и методы, разработанные в статье, могут быть непосредственно применены в синтезе пространственных регулируемых манипуляционных механизмов, предназначенных для приближённого воспроизведения заданных многоэтапных движений или множественных траекторий рабочего органа.

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