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On Convergence Acceleration of Trigonometric Interpolation

(Submitted by academician A. B. Nersessian 31/X 2011)

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1. In this paper we consider a convergence acceleration of the classical trigonometric interpolation

$$I_N(f,x)=\sum_{n=-N}^{N}\check{f}_n e^{i\pi nx}, \check{f}_n=\frac{1}{2N+1}\sum_{k=-N}^{N}f(x_k)e^{-i\pi nx_k}, x_k=\frac{2k}{2N+1}$$

via polynomial correction and along the ideas of rational (by $e^{i\pi x}$) approximations. Polynomial correction described in this paper is known as the Krylov-Lanczos approach (see [1-5] with references therein). The idea of convergence acceleration via rational functions corrections is described in [3, 6, 7]. We reveal some theoretical estimates for convergence of the suggested interpolations and discuss the problem of parameter determination.

In this section we introduce a rational interpolation as a method of convergence acceleration of the classical trigonometric interpolation. By $r_N(f,x)$ we denote the error of trigonometric interpolation

$$r_N(f,x)=f(x)-I_N(f,x).$$

We have

$$r_N(f,x)=\sum_{n=-N}^{N}\left(f_n-\check{f}_n\right)e^{i\pi nx}+\sum_{n=N+1}^{\infty}f_n e^{i\pi nx}+\sum_{n=-\infty}^{-N-1}f_n e^{i\pi nx},$$

where f_n is the n-th Fourier coefficient of f

$$f_n=\frac{1}{2}\int_{-1}^{1}f(x)e^{-i\pi nx}dx.$$

Consider a finite sequence of complex numbers $\theta=\{\theta_k\}_{|k|=1}^{p}$ and denote

$$\delta_n^0(\theta,c_n)=c_n,$$

$$\delta_n^k(\theta,c_n)=\delta_n^{k-1}(\theta,c_n)+\theta_{-k}\delta_{n-1}^{k-1}(\theta,c_n)+\theta_k\left(\delta_{n+1}^{k-1}(\theta,c_n)+\theta_{-k}\delta_n^{k-1}(\theta,c_n)\right).$$

By $\delta_n^k(c_n)$ we denote the sequence that corresponds to the choice $\theta\equiv 1$. It is easy to check that

$$\delta_n^k(c_n)=\Delta_{n+k}^{2k}(c_n)$$

where $\Delta_n^k(c_n)$ are the classical backward finite differences defined by the recurrence relation

$$\Delta_n^0(c_n)=c_n,\ \Delta_n^k(c_n)=\Delta_n^{k-1}(c_n)+\Delta_{n-1}^{k-1}(c_n).$$

We proceed by sequential application of the Abel type transformations. The following is easy to verify

$$r_N(f,x)=\breve{f}_N\theta_{-1}\frac{e^{-i\pi Nx}-e^{i\pi(N+1)x}}{\left(1+\theta_{-1}e^{i\pi x}\right)\left(1+\theta_1 e^{-i\pi x}\right)}+\breve{f}_{-N}\theta_{-1}\frac{e^{i\pi Nx}-e^{-i\pi(N+1)x}}{\left(1+\theta_{-1}e^{i\pi x}\right)\left(1+\theta_1 e^{-i\pi x}\right)}+$$
$$+\frac{1}{\left(1+\theta_{-1}e^{i\pi x}\right)\left(1+\theta_1 e^{-i\pi x}\right)}\sum_{|n|=N+1}^{\infty}\delta_n^1(\theta,f_n)e^{i\pi nx}$$
$$+\frac{1}{\left(1+\theta_{-1}e^{i\pi x}\right)\left(1+\theta_1 e^{-i\pi x}\right)}\sum_{n=-N}^{N}\delta_n^1(\theta,f_n-\breve{f}_n)e^{i\pi nx}.$$

Reiteration of this transformation up to p times yields to the following expansion of the error

$$r_N(f,x)=\left(e^{-i\pi Nx}-e^{i\pi(N+1)x}\right)\sum_{k=1}^{p}\frac{\theta_{-k}\delta_N^{k-1}(\theta,\breve{f}_n)}{\prod_{s=1}^{k}\left(1+\theta_{-s}e^{i\pi x}\right)\left(1+\theta_s e^{-i\pi x}\right)}$$
$$+\left(e^{i\pi Nx}-e^{-i\pi(N+1)x}\right)\sum_{k=1}^{p}\frac{\theta_k\delta_{-N}^{k-1}(\theta,\breve{f}_n)}{\prod_{s=1}^{k}\left(1+\theta_{-s}e^{i\pi x}\right)\left(1+\theta_s e^{-i\pi x}\right)}$$
$$+\frac{1}{\prod_{s=1}^{p}\left(1+\theta_{-s}e^{i\pi x}\right)\left(1+\theta_s e^{-i\pi x}\right)}\sum_{|n|=N+1}^{\infty}\delta_n^p(\theta,f_n)e^{i\pi nx}$$
$$+\frac{1}{\prod_{s=1}^{p}\left(1+\theta_{-s}e^{i\pi x}\right)\left(1+\theta_s e^{-i\pi x}\right)}\sum_{n=-N}^{N}\delta_n^p(\theta,f_n-\breve{f}_n),$$

where the first two terms in the right-hand side can be viewed as the corrections and the last two terms as the actual error. This viewing leads to the following rational-trigonometric interpolation

$$I_N^p(f,x)=I_N(f,x)+\left(e^{-i\pi Nx}-e^{i\pi(N+1)x}\right)\sum_{k=1}^{p}\frac{\theta_{-k}\delta_N^{k-1}(\theta,\breve{f}_n)}{\prod_{s=1}^{k}\left(1+\theta_{-s}e^{i\pi x}\right)\left(1+\theta_s e^{-i\pi x}\right)}$$
$$+\left(e^{i\pi Nx}-e^{-i\pi(N+1)x}\right)\sum_{k=1}^{p}\frac{\theta_k\delta_{-N}^{k-1}(\theta,\breve{f}_n)}{\prod_{s=1}^{k}\left(1+\theta_{-s}e^{i\pi x}\right)\left(1+\theta_s e^{-i\pi x}\right)}$$

with the error

$$r_N^p(f,x)=\frac{1}{\prod_{s=1}^{k}\left(1+\theta_{-s}e^{i\pi x}\right)\left(1+\theta_s e^{-i\pi x}\right)}\sum_{|n|=N+1}^{\infty}\delta_n^p(\theta,f_n)e^{i\pi nx}$$
$$+\frac{1}{\prod_{s=1}^{p}\left(1+\theta_{-s}e^{i\pi x}\right)\left(1+\theta_s e^{-i\pi x}\right)}\sum_{n=-N}^{N}\delta_n^p\left(\theta,f_n-\breve{f}_n\right)e^{i\pi nx}. \qquad (1)$$

Theorem 1. *Let $f\in C[-1,1]$. Then $I_N^p(f,x)$ is an interpolation of f on the equidistant grid $x_m=\dfrac{2m}{2N+1}$, $|m|\le N$ for every sequence θ with $|\theta_k|\ne 1$*

$$I_N^p(f,x_m)=f(x_m).$$

Proof. Let us first calculate the values of $e^{-i\pi Nx}-e^{i\pi(N+1)x}$ on the grid $x_m=\dfrac{2m}{2N+1}$

$$e^{-i\pi Nx_m}-e^{i\pi(N+1)x_m}=e^{-i\pi N\frac{2m}{2N+1}}-e^{i\pi(N+1)\frac{2m}{2N+1}}=e^{-i\pi m}e^{\frac{i\pi m}{2N+1}}-e^{i\pi m}e^{\frac{i\pi m}{2N+1}}=0.$$

Similarly

$$e^{i\pi Nx_m}-e^{-i\pi(N+1)x_m}=0.$$

Hence

$$I_N^p(f,x_m)=I_N(f,x_m)=f(x_m).$$

This completes the proof.

2. In this section we consider additional acceleration of the rational-trigonometric interpolation by polynomial correction method known as the Krylov-Lanczos approach.

Let $f\in C^{q-1}[-1,1]$. By $A_k(f)$ denote the jumps of f at the end points of the interval

$$A_k(f)=f^{(k)}(1)-f^{(k)}(-1),\ k=0,\dots,q-1.$$

The polynomial correction method is based on the following representation of the interpolated function

$$f(x)=\sum_{k=0}^{q-1}A_k(f)B_k(x)+F(x), \qquad (2)$$

where B_k are 2-periodic Bernoulli polynomials

$$B_0(x)=\frac{x}{2},B_k(x)=\int B_{k-1}(x)dx,\int_{-1}^{1}B_k(x)dx=0,x\in[-1,1]$$

and function F is a 2-periodic and relatively smooth function on the real line $\left(F\in C^{q-1}(R)\right)$ with the discrete Fourier coefficients

$$\breve{F}_n=\breve{f}_n-\sum_{k=0}^{q-1}A_k(f)\breve{B}_{k,n}.$$

Approximation of F in (2) by the classical trigonometric interpolation leads to the Krylov-Lanczos (KL-) interpolation

$$I_{N,q}(f,x)=\sum_{k=0}^{q-1}A_k(f)B_k(x)+I_N(F,x)$$

and approximation of F by the rational-trigonometric interpolation leads to rational-trigonometric-polynomial (rtp-) interpolation

$$I_{N,q}^{p}(f,x)=\sum_{k=0}^{q-1}A_k(f)B_k(x)+I_N^{p}(F,x)$$

with the errors $r_{N,q}(f,x)$ and $r_{N,q}^{p}(f,x)$, respectively.

Theorem 2. *Let* $f\in C^{q-1}[-1,1]$. *Then* $I_{N,q}^{p}(f,x)$ *is an interpolation of* f *on the equidistant grid* $x_m=\dfrac{2m}{2N+1}$, $|m|\le N$ *for every sequence* θ *with* $|\theta_k|\ne 1$

$$I_{N,q}^{p}(f,x_m)=f(x_m),\ |m|\le N.$$

Proof. In view of Theorem 1 and expansion (2) we get

$$\begin{aligned}I_{N,q}^{p}(f,x_m)&=\sum_{k=0}^{q-1}A_k(f)B_k(x_m)+I_N^{p}(F,x_m)\\&=\sum_{k=0}^{q-1}A_k(f)B_k(x_m)+F(x_m)=f(x_m).\end{aligned}$$

This completes the proof.

The next results we need for further comparisons. Theorem 3 describes the error of the KL-interpolation on the whole interval of interpolation in the framework of L_2-norm while Theorems 4 and 5 show the behavior of the pointwise-error in the regions away from the singularities $(x=\pm1)$.

Theorem 3. *[4] Let* $f\in C^{q}[-1,1]$ *and* $f^{(q)}\in AC[-1,1]$ *for some* $q\ge 1$. *Then the following estimate holds*

$$\left\|r_{N,q}(f,x)\right\|_{L_2}=O\left(N^{-q-\frac{1}{2}}\right),\ N\to\infty.$$

Theorem 4. *[4] Let* $q\ge 1$ *be an even number,* $f\in C^{q+1}[-1,1]$ *and* $f^{(q+1)}\in AC[-1,1]$. *Then the following estimate holds as* $N\to\infty$ *and* $|x|<1$ *is fixed*

$$r_{N,q}(f,x)=O\left(N^{-q-1}\right).$$

Theorem 5. *[4] Let* $q\ge 1$ *be an odd number,* $f\in C^{q+2}[-1,1]$ *and* $f^{(q+2)}\in AC[-1,1]$. *Then the following estimate holds as* $N\to\infty$ *and* $|x|<1$ *is fixed*

$$r_{N,q}(f,x)=O\left(N^{-q-2}\right).$$

3. Determination of parameter θ is crucial for realization of the rational-trigonometric interpolation. One general approach leads to *Fourier-Pade interpolation* and is connected with the solution of the following system for getting the values of θ_k

$$\delta_n^p\left(\theta, \breve{f}_n\right) = 0, \ |n| = N - p + 1, \ldots, N,$$

where the periodicity property $\breve{f}_{n+k(2N+1)} = \breve{f}_n$ of the discrete Fourier coefficients is taken into account. Theoretical investigation of such interpolations will be carried out elsewhere.

Here we consider another choice of parameter θ

$$\theta_k = \theta_{-k} = 1 - \frac{\tau_k}{N}, k = 1, \ldots, p, \tau_k > 0, \tau_j \neq \tau_i, j \neq i \tag{3}$$

and introduce theoretical estimates for the corresponding interpolations for a smooth function f on $[-1,1]$. New parameters τ_k can be determined by different approaches. One approach leads to L_2-minimal interpolation. This idea is introduced and investigated in [3] for the Fourier-Pade approximations. The first step towards L_2-minimal interpolation is performed in [6] where the case $p = 1$ is investigated. The idea of this interpolation is determination of unknown parameters τ_k from the condition

$$\lim_{N \to \infty} N^{q+\frac{1}{2}} \left\| r_{N,q}^p(f, x) \right\|_{L_2} \to \text{minimum}.$$

Paper [6] shows the solution of this problem for $p = 1$ and $1 \le q \le 6$. Similarly other cases can be investigated.

Another approach for determination of parameters τ_k is described in [8] where τ_k are the roots (which are positive and distinct) of the Laguerre polynomial $L_p^q(x)$.

The rest of the paper is devoted to the derivation of the analogs of Theorems 3, 4 and 5 for the rtp-interpolation for smooth functions on $[-1,1]$ where parameter θ is defined as in (3).

The following theorem shows the convergence rate of the rtp-interpolation in the regions away from the singularities $x = \pm 1$.

Theorem 6. *Let $f \in C^{q+2p+1}[-1,1]$ and $f^{(q+2p+1)} \in AC[-1,1]$ for some $p,q \ge 1$ and parameters θ are chosen as in (3). Then the following estimate holds as $N \to \infty$ and $|x| < 1$ is fixed*

$$r_{N,q}^p(f,x) = O\left(N^{-q-2p-1}\right).$$

Proof. Expansion (2) shows that

$$r_{N,q}^p(f,x) = r_N^p(F,x).$$

We get by the Abel transformation (see (1))

$$c(x) r_{N,q}^p(f,x) = \delta_N^p\left(\theta, \breve{F}_n\right)\left(e^{-i\pi Nx} - e^{i\pi(N+1)x}\right) + \delta_{-N}^p\left(\theta, \breve{F}_n\right)\left(e^{i\pi Nx} - e^{-i\pi(N+1)x}\right) +$$

$$+\sum_{n=-N}^{N}\delta_n^1\left(\delta_n^p\left(\theta,F_n-\breve{F}_n\right)\right)e^{i\pi nx}+\sum_{|n|=N+1}^{N}\delta_n^1\left(\delta_n^p\left(\theta,F_n\right)\right)e^{i\pi nx}$$

(4)
where

$$c=4\cos^2\frac{\pi x}{2}\prod_{s=1}^{p}\left(1+2\theta_s\cos\pi x+\theta_s^2\right).$$

First we will estimate the last term in the right-hand side of (4). We need to estimate $\delta_n^1\left(\delta_n^p\left(\theta,F_n\right)\right)$ for $|n|>N$. In view of definitions of $\delta_n^k\left(\theta,c_n\right)$, $\delta_n^k\left(c_n\right)$ and their connection with $\Delta_n^k\left(c_n\right)$ we get (see also (3))

$$\delta_n^1\left(\delta_n^p\left(\theta,F_n\right)\right)=\sum_{s=0}^{p}(-1)^s\frac{\gamma_s}{N^s}\sum_{k=0}^{p}(-1)^k\frac{\gamma_k}{N^k}\Delta_{n+p-s+1}^{2p-k-s+2}\left(F_n\right),$$

where γ_k are the coefficients of polynomial

$$\prod_{s=1}^{p}\left(1+\tau_s\,x\right)=\sum_{s=0}^{p}\gamma_s\,x^s.$$

In view of the smoothness of F we get (by means of integration by parts and from expansion (2))

$$F_n=\sum_{m=q}^{q+2p+1}A_m\left(f\right)B_{m,n}+\frac{1}{2\left(i\pi n\right)^{q+2p+2}}\int_{-1}^{1}f^{(q+2p+2)}\left(x\right)e^{-i\pi nx}dx. \tag{6}$$

Taking into account that the last term in (6) is $\frac{o(1)}{n^{q+2p+2}}$, $N\to\infty$ and the well-known estimate (see [4] for similar estimates)

$$\Delta_n^k\left(B_{m,n}\right)=O\left(\frac{1}{n^{m+k+1}}\right),\ n\to\infty$$

we obtain

$$\Delta_{n+p-s+1}^{2p-k-s+2}\left(F_n\right)=\sum_{m=q}^{q+2p+1}O\left(\frac{1}{n^{m+2p-k-s+3}}\right)+\frac{o(1)}{n^{q+2p+2}},\ n\to\infty.$$

Substituting this into (5) we obtain

$$\delta_n^1\left(\delta_n^p\left(\theta,F_n\right)\right)=\frac{1}{N^{2p}}o\left(n^{-q-2}\right),\ |n|>N,\ N\to\infty$$

From here we conclude that the last term in the right-hand side of (4) is $o\left(N^{-q-2p-1}\right)$ as $N\to\infty$.

Now we will estimate the third term in the right-hand side of (4). We need to estimate $\delta_n^1\left(\delta_n^p\left(\theta,F_n-\breve{F}_n\right)\right)$ for $|n|\le N$. Similar to (5) we write

$$\delta_n^1\left(\delta_n^p\left(\theta,F_n-\breve{F}_n\right)\right)=\sum_{s=0}^{p}(-1)^s\frac{\gamma_s}{N^s}\sum_{k=0}^{p}(-1)^k\frac{\gamma_k}{N^k}\Delta_{n+p-s+1}^{2p-k-s+2}\left(F_n-\breve{F}_n\right). \tag{7}$$

Discrete Fourier coefficient $\breve{F}_n$ is convenient to estimate based on the identities

$$\breve{F}_n=\sum_{s=-\infty}^{\infty}F_{n+s(2N+1)}\ \text{and}\ \breve{F}_n-F_n=\sum_{s\neq\infty}F_{n+s(2N+1)}. \tag{8}$$

Applying expansion (6) we obtain for $|n| \le N$

$$\breve{F}_n - F_n = \sum_{m=q}^{q+2p+1} A_m(f) \sum_{s \ne 0} B_{m,n+s(2N+1)} + \frac{o(1)}{N^{q+2p+2}}, N \to \infty .$$

Using the estimate (see [4] for similar estimates)

$$\Delta_n^k \left(\sum_{s \ne 0} B_{m,n+s(2N+1)} \right) = O\left(N^{-m-k-1}\right), \ N \to \infty$$

we get

$$\Delta_{n+p-s+1}^{2p-k-s+2} \left(F_n - \breve{F}_n \right) = O\left(\frac{1}{n^{2p-k-s+q+3}} \right) + \frac{o(1)}{n^{q+2p+2}}, \ N \to \infty .$$

Substituting this into (7) we obtain

$$\delta_n^1 \left(\delta_n^p \left(\theta, F_n - \breve{F}_n \right) \right) = \frac{o(1)}{N^{q+2+2}}, \ |n| \le N, N \to \infty .$$

Hence, the third term in the right-hand side of (4) is $o\left(N^{-q-2p-1}\right)$ as $N \to \infty$.

Finally, we need to estimate the first two terms in the right-hand side of (4). We need to estimate $\delta_{\pm N}^p \left(\theta, \breve{F}_n \right)$. Similar to (5) we write

$$\delta_{\pm N}^p \left(\theta, \breve{F}_n \right) = \sum_{s=0}^{p} (-1)^s \frac{\gamma_s}{N^s} \sum_{k=0}^{p} (-1)^k \frac{\gamma_k}{N^k} \Delta_{\pm N+p-s}^{2p-k-s} \left(\breve{F}_n \right). \tag{9}$$

In view of (6) and (8) we have (see similar estimate in [4])

$$\Delta_{\pm N}^k \left(\breve{F}_n \right) = O\left(\frac{1}{N^{q+k+1}} \right), \ N \to \infty .$$

Therefore

$$\Delta_{\pm N+p-s}^{2p-k-s} \left(\breve{F}_n \right) = O\left(\frac{1}{N^{q+2p-k-s+1}} \right), \ N \to \infty$$

and

$$\delta_{\pm N}^p \left(\theta, \breve{F}_n \right) = O\left(\frac{1}{N^{q+2p+1}} \right), N \to \infty$$

which completes the proof.

Next theorem is analog of Theorem 3. We omit the proof as it mimics the proof of similar theorem in [3].

Theorem 7. *Let* $f \in C^{q+2p}[-1,1]$ *and* $f^{(q+2p+1)} \in AC[-1,1]$ *for some* $q, p \ge 1$ *and parameter* θ *chosen as in (3). Then the following estimate holds*

$$\left\| r_{N,q}^p (f,x) \right\|_{L_2} = O\left(N^{q+1/2}\right), N \to \infty .$$

4. In this section we compare the convergence of the KL and rtp interpolations in the regions away from the singularities. Parameter θ we take as in (3) and as τ_k we put the roots of the Laguerre polynomials $L_p^q(x)$.

It is important to notice that theorems for rtp-interpolation require additional smoothness from the approximated functions and in comparisons it must be taken into account. If q is the number of available jumps and $p > 0$ is chosen such that the requirements of Theorem 6 are valid (e.g. when function is infinitely differentiable) then rtp-interpolation is more precise (however asymptotically) than the KL-interpolation which follows from comparison of Theorems 4, 5 and 6. Also we will show that utilization of all available jumps is not always reasonable and more accuracy can be achieved with less jumps in combination with rational corrections.

Let $f \in C^{M+1}[-1,1]$ and $f^{(M+1)} \in AC[-1,1]$, $M \geq 1$. Let q be even. According to Theorems 4 and 6 if the values of p and q satisfy the condition $q+2p=M$ then both theorems are valid and comparison of corresponding approximations is legal. Let

$$f(x)=\sin(ax-1)$$

where a is some parameter. We use the values $a=1,10,30$ and in Table 1 calculate the values of $\max_{x\in[-0.5,0.5]}\left|r_{N,q}^{p}(f,x)\right|$ for $N=512$ and for different values of p and q with condition $q+2p=6$.

Table 1. Values of $\max_{x\in[-0.5,0.5]}\left|r_{N,q}^{p}(f,x)\right|$ for $N=512$, $q+2p=6$, $q=2,4$ and 6 and

$$f(x)=\sin(ax-1)$$

	$q=2$ $p=2$	$q=4$ $p=1$	$q=6$ $p=0$
$a=1$	$2.0\cdot10^{-20}$	$1.1\cdot10^{-21}$	$4.6\cdot10^{-23}$
$a=10$	$1.3\cdot10^{-18}$	$6.9\cdot10^{-18}$	$3.0\cdot10^{-17}$
$a=30$	$2.1\cdot10^{-17}$	$1.0\cdot10^{-15}$	$3.9\cdot10^{-14}$

The table shows that for $a=1$ the KL-interpolation $S_{N,6}(f,x)$ is the best, for $a=10$ and $a=30$ the rtp-interpolation $S_{N,2}^{2}(f,x)$ is the best. For $a=30$ the rtp-interpolation is much more accurate than KL-interpolation which uses all available jumps.

Actually we came to the conclusion (not only based on this specific example but also on the expansion (6) where the main terms include jumps $A_q(f), A_{q+1}(f),\ldots$ that not always utilization of all available jumps, by the KL-interpolation, leads to the best approximation. When the values of jumps are rapidly increasing then better accuracy can be achieved by utilization of smaller number of jumps and appropriately chosenrational corrections.

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On Convergence Acceleration of Trigonometric Interpolation

A method of convergence acceleration of the classical trigonometric interpolation is considered, which leads to interpolation with rational functions with the unknown parameter. Some convergence theorems regarding the special choice of this parameter are presented.

Ա. Վ. Պողոսյան
Եռանկյունաչափական ինտերպոլյացիայի զուգամիտության արագացման մասին

Ուսումնասիրվում է եռանկյունաչափական ինտերպոլյացիայի զուգամիտության արագացման մի եղանակ, որի արդյունքում ստացվում է ռացիոնալ ֆունկցիաներով իրականացվող ինտերպոլյացիա՝ կախված անհայտ պարամետրից: Ներկայացվում են զուգամիտության թեորեմներ՝ այդ պարամետրի մի մասնավոր ընտրության համար:

А. В. Погосян

Об ускорении сходимости тригонометрической интерполяции

Рассматривается метод ускорения сходимости тригонометрической интерполяции, который приводит к интерполяции с рациональными функциями с неизвестным параметром. Приводятся теоремы сходимости для одного выбора этого параметра.

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