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On a Convergence of the L_2-Optimal Rational Approximation

(Submitted by academician A. B. Nersessian 31/X 2011)

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1.We consider the problem of a function approximation by finite number of its Fourier coefficients

$$f_n = \frac{1}{2}\int_{-1}^{1} f(x) e^{-i\pi nx} dx, \ |n| \le N.$$

It is well known that approximation of a 2-periodic and smooth function f on the real line by the truncated Fourier series

$$S_N(f) = \sum_{n=-N}^{N} f_n e^{i\pi nx}$$

is highly effective. When the approximated function has a point of discontinuety, approximation by the partial sum $S_N(f)$ leads to the Gibbs phenomenon.

In this paper we consider the L_2-optimal rational approximation in combination with the Krylov-Lanczos approximation and investigate a convergence of this approach in the regions where the approximated function is smooth.

Denote

$$R_N(f) = f(x) - S_N(f).$$

2. First we introduce the Krylov-Lanczos approximation (see [1-3]). Suppose $f \in C^q[-1,1]$. By $A_k(f)$ denote the exact value of the jump in the k-th derivative of f

$$A_k(f) = f^{(k)}(1) - f^{(k)}(-1), \ k = 0,\dots,q.$$

The basic idea of the Krylov-Lanczos approximation is the following representation of the approximated function

$$f(x) = F(x) + \sum_{k=0}^{q-1} A_k(f) B_k(x),$$

where B_k are 2-periodic Bernoulli polynomials with the Fourier coefficients

$$B_{k,n} = \begin{cases} 0, & n=0 \\ \dfrac{(-1)^{n+1}}{2(i\pi n)^{k+1}}, & n \neq 0 \end{cases}$$

and F is a 2-periodic and relatively smooth function on the real line $(F \in$ $\in C^{q-1}(R))$ with the Fourier coefficients

$$F_n = f_n - \sum_{k=0}^{q-1} A_k(f) B_{k,n}. \qquad (1)$$

Approximation of F by the truncated Fourier series leads to the Krylov-Lanczos (KL-) approximation

$$S_{N,q}(f) = \sum_{n=-N}^{N} F_n e^{i\pi nx} + \sum_{k=0}^{q-1} A_k(f) B_k(x)$$

with the error

$$R_{N,q}(f) = f(x) - S_{N,q}(f).$$

By $\|\cdot\|$ denote the standard norm in the space L_2

$$\|f\| = \left(\int_{-1}^{1} |f(x)|^2 dx\right)^{1/2}.$$

The following results we need for further comparison.

Theorem 1 [3]. *Suppose* $f \in C^q[-1,1]$ *and* $f^{(q)} \in AC[-1,1]$. *Then the following estimate holds*

$$\lim_{N\to\infty} N^{q+\frac{1}{2}} \|R_{N,q}(f)\| = |A_q(f)| c(q),$$

where

$$c(q) = \frac{1}{\pi^{q+1}\sqrt{2q+1}}.$$

Theorem 2 [4]. *Suppose* $f \in C^{q+1}[-1,1]$ *and* $f^{(q+1)} \in AC[-1,1]$. *Then the following estimates hold for* $|x|<1$

$$R_{N,q}(f) = A_q(f)\frac{\varphi_{N,q}(x)}{N^{q+1}} + o(N^{-q-1}), \ N \to \infty,$$

where

$$\varphi_{N,q}(x) = \frac{(-1)^{N+\frac{q}{2}}}{2\pi^{q+1}} \frac{\sin\frac{\pi x}{2}(2N+1)}{\cos\frac{\pi x}{2}}$$

for even values of q *and*

$$\varphi_{N,q}(x) = \frac{(-1)^{N+\frac{q+1}{2}}}{2\pi^{q+1}} \frac{\cos\frac{\pi x}{2}(2N+1)}{\cos\frac{\pi x}{2}}$$

for odd values of q.

3. Now we introduce the L_2-optimal rational approximation for additional convergence acceleration of the KL-approximation (see [4]).

Consider a finite sequence of complex numbers $\theta = \{\theta_k\}_{|k|=1}^{p}$, $p \ge 1$ and denote

$$\Delta_n^0(\theta, F_n) = F_n, \Delta_n^k(\theta, F_n) = \Delta_n^{k-1}(\theta, F_n) + \theta_{k\,\mathrm{sgn}(n)} \Delta_{(|n|-1)sqn(n)}^{k-1}(\theta, F_n), k \ge 1,$$

where $\mathrm{sgn}(n) = 1$ if $n \ge 0$ and $\mathrm{sgn}(n) = -1$ if $n < 0$.

By $\Delta_n^p(F_n)$ we denote the classical finite differences which correspond to generalized differences $\Delta_n^p(\theta, F_n)$ with $\theta \equiv 1$.

We have

$$R_{N,q}(f) = R_N^+(F) + R_N^-(F),$$

where

$$R_N^+(F) = \sum_{n=N+1}^{\infty} F_n e^{i\pi nx}, \quad R_N^-(F) = \sum_{n=-\infty}^{-N-1} F_n e^{i\pi nx}.$$

It can easily be checked that

$$R_N^+(F) = -\frac{\theta_1 F_N e^{i\pi(N+1)x}}{1+\theta_1 e^{i\pi x}} + \frac{1}{1+\theta_1 e^{i\pi x}} \sum_{n=N+1}^{\infty} \Delta_n^1(\theta, F_n) e^{i\pi nx}.$$

Reiteration of this transformation up to p times leads to the following expansion

$$R_N^+(F) = -e^{i\pi(N+1)x} \sum_{k=1}^{p} \frac{\theta_k \Delta_N^{k-1}(\theta, F_n)}{\prod_{s=1}^{k}\left(1+\theta_s e^{i\pi x}\right)} + \frac{1}{\prod_{k=1}^{p}\left(1+\theta_k e^{i\pi x}\right)} \sum_{n=N+1}^{\infty} \Delta_n^p(\theta, F_n) e^{i\pi nx}.$$

Similar expansion of $R_N^-(F)$ reduces to the following rational (by $e^{i\pi nx}$) approximation

$$S_{N,q,p}(f) = \sum_{k=0}^{q-1} A_k(f) B_k(x) + \sum_{n=-N}^{N} F_n e^{i\pi nx}$$
$$-e^{i\pi(N+1)x} \sum_{k=1}^{p} \frac{\theta_k \Delta_N^{k-1}(\theta, F_n)}{\prod_{s=1}^{k}\left(1+\theta_s e^{i\pi x}\right)} - e^{-i\pi(N+1)x} \sum_{k=1}^{p} \frac{\theta_{-k} \Delta_{-N}^{k-1}(\theta, F_n)}{\prod_{s=1}^{k}\left(1+\theta_{-s} e^{-i\pi x}\right)}$$

with the error

$$R_{N,q,p}(f) = f(x) - S_{N,q,p}(f) = R_{N,q,p}^+(f) + R_{N,q,p}^-(f),$$

where

$$R_{N,q,p}^{\pm}(f) = \frac{1}{\prod_{k=1}^{p}\left(1+\theta_{\pm k} e^{\pm i\pi x}\right)} \sum_{n=N+1}^{\infty} \Delta_{\pm n}^p(\theta, F_n) e^{\pm i\pi nx}. \qquad (2)$$

Different methods are known for determination of the unknown parameter θ. One method is described in [4] which leads to the L_2-optimal rational approximation. In particular, parameters θ_k and θ_{-k} are chosen as follows (see Theorem 3)

$$\theta_k=\theta_{-k}=1-\frac{\tau_k}{N}, k=1,\ldots,p, \tau_k>0, \tau_j\neq\tau_i, j\neq i,$$

where the new parameters τ_k minimize the constant $c_p(q)$ in Theorem 3.

By $\gamma_k(p)$ denote the coefficients of the polynomial

$$\prod_{k=1}^{p}(1+\tau_k x)=\sum_{k=0}^{p}\tau_k(p)x^k.$$

Theorem 3 [4]. Suppose $f\in C^{q+p}[-1,1]$ and $f^{(q+p)}\in AC[-1,1]$. If

$$\theta_k=\theta_{-k}=1-\frac{\tau_k}{N}, k=1,\ldots,p, \tau_k>0, \tau_j\neq\tau_i, j\neq i,$$

then the following estimate holds

$$\lim_{N\to\infty}N^{q+\frac{1}{2}}\left\|R_{N,q,p}(f)\right\|=\left|A_q(f)\right|c_p(q),$$

where

$$c_p(q)=\frac{1}{\pi^{q+1}}\left(\int_1^{\infty}\left|\phi_{p,q}(t)\right|^2dt\right)^{1/2},$$

and

$$\phi_{p,q}(t)=\frac{(-1)^p}{t^{q+1}}-\frac{1}{q!}\sum_{j=1}^{p}\frac{e^{-\tau_j(t-1)}}{\prod_{\substack{i=1\\ i\neq j}}^{p}(\tau_i-\tau_j)}\sum_{k=0}^{p}\gamma_k(p)(-1)^{k+1}\sum_{m=0}^{p-k-1}(q+p-k-m-1)!\tau_j^m .$$

Table 1 presents the numerical values of $c_p(q)$ by using parameters τ_k that minimize it. Note that $c_0(q)=c(q)$.

Table 1

Numerical values of $c(q)$ and $c_p(q)$ by using the optimal values of parameters

q	1	2	3	4	5	6
$c(q)$	$5.8\cdot10^{-2}$	$1.4\cdot10^{-2}$	$3.9\cdot10^{-3}$	$1.1\cdot10^{-3}$	$3.1\cdot10^{-4}$	$9.2\cdot10^{-5}$
$c_1(q)$	$1.0\cdot10^{-2}$	$1.6\cdot10^{-3}$	$3.2\cdot10^{-4}$	$7.0\cdot10^{-5}$	$1.7\cdot10^{-5}$	$4.2\cdot10^{-2}$
$c_2(q)$	$2.8\cdot10^{-3}$	$3.1\cdot10^{-4}$	$4.7\cdot10^{-5}$	$8.5\cdot10^{-6}$	$1.7\cdot10^{-6}$	$3.7\cdot10^{-7}$
$c_3(q)$	$9.5\cdot10^{-4}$	$7.8\cdot10^{-5}$	$9.4\cdot10^{-6}$	$1.4\cdot10^{-6}$	$2.4\cdot10^{-7}$	$4.6\cdot10^{-8}$
$c_4(q)$	$3.7\cdot10^{-4}$	$2.3\cdot10^{-5}$	$2.3\cdot10^{-6}$	$2.9\cdot10^{-7}$	$4.3\cdot10^{-8}$	$7.0\cdot10^{-9}$
$c_5(q)$	$1.6\cdot10^{-4}$	$7.8\cdot10^{-6}$	$6.3\cdot10^{-7}$	$6.8\cdot10^{-8}$	$8.7\cdot10^{-9}$	$1.3\cdot10^{-9}$

4. In this section we investigate the pointwise convergence of approximation $S_{N,q,p}(f)$ in the regions away from the singularities ($|x|<1$).

The main result of this paper is:

Theorem 4. Let $f\in C^{q+p+1}[-1,1]$ *and* $f^{(q+p+1)}\in AC[-1,1]$. If

$$\theta_k = \theta_{-k} = 1 - \frac{\tau_k}{N}$$

then the following estimates hold for $|x| < 1$

$$R_{N,q,p}(f) = A_q(f)\frac{\varphi_{N,q,p}(x)}{N^{q+p+1}} + o\left(N^{-q-p-1}\right), N \to \infty,$$

where

$$\varphi_{N,q,p}(x) = \frac{(-1)^{N+p+\frac{q}{2}}}{2^{p+1}\pi^{q+1}q!} \frac{\sin\frac{\pi x}{2}(2N-p+1)}{\cos^{p+1}\frac{\pi x}{2}} \sum_{k=0}^{p} (-1)^k (p-k+q)!\gamma_k(p)$$

for even values of q and

$$\varphi_{N,q,p}(x) = \frac{(-1)^{N+p+\frac{q+1}{2}}}{2^{p+1}\pi^{q+1}q!} \frac{\cos\frac{\pi x}{2}(2N-p+1)}{\cos^{p+1}\frac{\pi x}{2}} \sum_{k=0}^{p} (-1)^k (p-k+q)!\gamma_k(p)$$

for odd values of q.

Proof. Taking into account that $\theta_k \to 1$ as $N \to \infty$ then we need to estimate only the last sum in (2). By the Abel transformation we get

$$\begin{aligned}\sum_{n=N+1}^{\infty} \Delta_{\pm n}^{p}(\theta, F_n) e^{\pm i\pi nx} = &-\frac{e^{\pm i\pi(N+1)x}}{1+e^{\pm i\pi x}} \Delta_{\pm N}^{0}\left(\Delta_n^p(\theta, F_n)\right) \\ &-\frac{e^{\pm i\pi(N+1)x}}{\left(1+e^{\pm i\pi x}\right)^2} \Delta_{\pm N}^{1}\left(\Delta_n^p(\theta, F_n)\right) \\ &+\frac{1}{\left(1+e^{\pm i\pi x}\right)^2} \sum_{n=N+1}^{\infty} \Delta_{\pm n}^{2}\left(\Delta_n^p(\theta, F_n)\right) e^{\pm i\pi nx}.\end{aligned} \tag{3}$$

Now we estimate the sequences $\Delta_{\pm N}^{0}\left(\Delta_n^p(\theta, F_n)\right)$, $\Delta_{\pm N}^{1}\left(\Delta_n^p(\theta, F_n)\right)$ and $\Delta_{\pm n}^{2}\left(\Delta_n^p(\theta, F_n)\right)$ as $N \to \infty$ and $n \geq N+1$.

It is easy to verify that

$$\Delta_n^p(\theta, F_n) = \sum_{k=0}^{p} \frac{(-1)^k \gamma_k(p)}{N^k} \Delta_{n-\mathrm{sgn}(n)k}^{p-k}(F_n),$$

where the classical finite differences can be calculated by the formula

$$\Delta_n^k(F_n) = \sum_{j=0}^{p} \binom{k}{j} F_{n-\mathrm{sgn}(n)j}.$$

Taking into account that $\Delta_n^w\left(\Delta_n^{p-k}(F_n)\right) = \Delta_n^{w+p-k}(F_n)$, we get

$$\Delta_n^w\left(\Delta_n^p(\theta, F_n)\right) = \sum_{k=0}^{p} \frac{(-1)^k \gamma_k(p)}{N^k} \sum_{j=0}^{w+p-k} \binom{w+p-k}{j} F_{n-\mathrm{sgn}(n)(k+j)}. \tag{4}$$

Smoothness of F leads to the following asymptotic expansions of the Fourier coefficients

$$F_n=\frac{(-1)^{n+1}}{2}\sum_{s=q}^{q+p-k+1}\frac{A_s(f)}{(i\pi n)^{s+1}}+o\left(n^{-q-p+k+2}\right),\ n\to\infty,\ k=0,\ldots,p.$$

Substituting this into (4), we derive

$$\Delta_n^w\left(\Delta_n^p(\theta,F_n)\right)=\frac{(-1)^{n+1}}{2}\sum_{k=0}^{p}\frac{\gamma_k(p)}{N^k}\sum_{j=0}^{w+p-k}(-1)^j\binom{w+p-k}{j}\sum_{s=q}^{q+p-k+1}\frac{A_s(f)}{(i\pi n)^{s+1}}$$

$$\times\frac{1}{\left(1-\frac{\pm(k+j)}{n}\right)^{s+1}}+o\left(n^{-q-p-2}\right)$$

$$=\frac{(-1)^{n+1}}{2(i\pi n)^{q+1}}\sum_{k=0}^{p}\frac{\gamma_k(p)}{N^k}\sum_{t=0}^{p-k+1}\frac{1}{n^t}\sum_{s=0}^{t}(\pm1)^s\binom{t+q}{s}\frac{A_{t-s+q}(f)}{(i\pi)^{t-s}}\alpha_{k,s}(w)+o\left(n^{-q-p-2}\right),$$

where

$$\alpha_{k,s}(w)=\sum_{j=0}^{w+p-k}(-1)^j\binom{w+p-k}{j}(k+j)^s$$

and "+" sign corresponds to positive values of n and "-" sign to negative values. It is well known [4] that $\alpha_{k,s}(w)=0$ for $s<w+p-k$, hence for $s\ge w+p-k$ and consenquently $t\ge w+p-k$, after simple manipulations, we derive

$$\Delta_n^w\left(\Delta_n^p(\theta,F_n)\right)=\frac{(-1)^{n+1}}{2(i\pi n)^{q+1}}\sum_{k=0}^{p}\frac{\gamma_k(p)}{N^k}\sum_{t=w}^{1}\frac{\beta_{t,k}^{\pm}(w)}{n^{p-k+t}}+o\left(n^{-q-p-2}\right),\tag{5}$$

where

$$\beta_{t,k}^{\pm}(w)=\sum_{s=w}^{t}(\pm1)^{p+s-k}\binom{t+p-k+q}{p-k+s}\frac{A_{t-s+q}(f)}{(i\pi)^{t-s}}\alpha_{k,s+p-k}(w).$$

Asymptotic expansion (5) immediately shows that $\Delta_n^2\left(\Delta_N^p(\theta,F_n)\right)=o\left(n^{-p-q-2}\right)$ and the third term in the right hand side of (3) is $o\left(N^{-q-p-1}\right)$ as $N\to\infty$.

Taking $n=\pm N$ in (5), we get

$$\Delta_{\pm N}^w\left(\Delta_n^p(\theta,F_n)\right)=\frac{(-1)^{N+1}}{2(\pm i\pi N)^{q+1}N^p}\sum_{k=0}^{p}\gamma_k(p)\sum_{t=w}^{1}\frac{(\pm1)^{t+p-k}\beta_{t,k}^{\pm}(w)}{N^t}+o\left(n^{-q-p-2}\right).\tag{6}$$

From here we conclude that $\Delta_{\pm N}^1\left(\Delta_n^p(\theta,F_n)\right)=O\left(N^{-p-q-2}\right)$ as $N\to\infty$. Then

$$\Delta_{\pm N}^0\left(\Delta_n^p(\theta,F_n)\right)=\frac{(-1)^{N+1}}{2(\pm i\pi N)^{q+1}N^p}\sum_{k=0}^{p}(\pm1)^{p-k}\gamma_k(p)\beta_{0,k}^{\pm}(0)+O\left(N^{-p-q-2}\right).$$

Taking into account the relations

$$(\pm1)^{p-k}\beta_{0,k}^{\pm}(0)=A_q(f)\binom{p-k+q}{p-k}\alpha_{k,p-k}(0)$$

$$=A_q(f)\binom{p-k+q}{p-k}\sum_{j=0}^{p-k}(-1)^j\binom{p-k}{j}(k+j)^{p-k}$$

$$= A_q(f)(-1)^{p-k}\frac{(p-k+q)!}{q!}$$

we get

$$\Delta^0_{\pm N}\left(\Delta^p_n(\theta,F_n)\right) = A_q(f)\frac{(-1)^{N+p+1}}{2(\pm i\pi N)^{q+1}N^p q!}\sum_{k=0}^{p}(-1)^k (p-k+q)!\gamma_k(p)+O\left(N^{-p-q-2}\right).$$

Now it follows from (3) that

$$R^{\pm}_{N,q,p}(f) = \frac{e^{\pm i\pi(N+1)x}}{\left(1+e^{\pm i\pi x}\right)^{p+1}}\frac{(-1)^{N+p}}{2N^{p+q+1}q!}\frac{A_q(f)}{(\pm i\pi)^{q+1}}\sum_{k=0}^{p}(-1)^k (p-k+q)!\gamma_k(p)+o\left(N^{-p-q-1}\right).$$

Finally

$$R_{N,q,p}(f) = \frac{(-1)^{N+p}}{N^{p+q+1}q!}\frac{A_q(f)}{\pi^{q+1}}\sum_{k=0}^{p}(-1)^k (p-k+q)!\gamma_k(p)$$

$$\mathrm{Re}\left[\frac{e^{i\pi(N+1)x}}{i^{q+1}\left(1+e^{i\pi x}\right)^{p+1}}\right]+o\left(N^{-p-q-1}\right).$$

This completes the proof.

5. In this section we compare the convergence of the KL and L_2-optimal rational approximations in the regions away from the singularities and show how the parameters p and q can be chosen in practice for better approximation. We will show that utilization of all available jumps is not always reasonable and more accuracy can be achieved with less jumps in combination with rational corrections.

It is important to notice that Theorem 4 puts additional smoothness requirements on the approximated functions compared to Theorem 2 so in comparisons it must be taken into account. If q is the number of available jumps and $p>0$ is chosen such that the requirements of Theorem 4 are valid (e.g. when function is infinitely differentiable) then L_2-optimal rational approximation is more precise (however asymptotically) than the KL-approximation which follows from comparison of Theorems 2 and 4.

Let $f\in C^{M+1}[-1,1]$, $f^{(M+1)}\in AC[-1,1]$, $M\geq 1$. According to Theorems 2 and 4 if the values of q and q satisfy the condition $q+p=M$ then both Theorems 2 and 4 are valid and comparison of corresponding approximations is legal. Then, asymptotic estimates of these theorems will show which values of parameters p and q provide with better accuracy. We show this process for a specific example. Let

$$f(x)=\sin(ax-1),$$

where a is some parameter. We use the values $a = 1/5, 3, 50$ and in Table 2 calculate the values of $\frac{|A_q(f)|}{N^{q+p+1}} \max_{x\in[-0.5,0.5]} |\varphi_{N,q,p}(x)|$ for $N = 512$ and for different values of p and q with condition $q + p = 5$. Recall that $p = 0$ corresponds to the KL-approximation.

Table 2

Values of $\frac{|A_q(f)|}{N^{q+p+1}} \max_{x\in[-0.5,0.5]} |\varphi_{N,q,p}(x)|$ for $N = 512$, $q + p = 5$, $p = 0,\ldots,4$ and $f(x) = \sin(ax-1)$

	$q=5$ $p=0$	$q=4$ $p=1$	$q=3$ $p=2$	$q=2$ $p=3$	$q=1$ $p=4$
$a=1/5$	$4.2\cdot10^{-24}$	$2.2\cdot10^{-23}$	$5.1\cdot10^{-22}$	$7.7\cdot10^{-21}$	$2.4\cdot10^{-19}$
$a=3$	$2.3\cdot10^{-18}$	$7.9\cdot10^{-19}$	$1.2\cdot10^{-18}$	$1.2\cdot10^{-18}$	$2.6\cdot10^{-18}$
$a=50$	$5.5\cdot10^{-12}$	$1.1\cdot10^{-13}$	$1.1\cdot10^{-14}$	$6.4\cdot10^{-16}$	$8.0\cdot10^{-17}$

The table shows that for $a = 1/5$ the KL-approximation $S_{N,5}(f)$ is the best, for $a = 3$ the L_2-optimal rational approximation $S_{N,4,1}(f)$ is the best and for $a = 50$ the approximation $S_{N,1,4}(f)$ is the best.

Overall conclusion based on this specific example and on comparison of the asymptotic estimates of Theorems 2 and 4 is the following: not always utilization of all available jumps, by the KL-approximation, leads to the best approximation. This is due to the factor $A_q(f)$ in the estimates. When the values of jumps are rapidly increasing then better accuracy can be achieved by utilization of smaller number of jumps and appropriately chosen corrections based on the smoothness of the approximated function. Which choice of q and p is the best can be concluded from comparison of the corresponding estimates as we did above.

It must be also mentioned that when the jumps are rapidly increasing then getting their approximations is problematic so in that case utilization of the rational corrections is unavoidable for better accuracy.

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On a Convergence of the L_2-Optimal Rational Approximation

We investigate a convergence of the L_2-optimal rational approximation in the regions away from singularities where the approximated function is smooth. Theoretical estimates show that the rate of convergence is greater than for the classical Krylov-Lanczos method by the order which equals to the order of denominator of the rational approximants.

Ա. Վ. Պողոսյան

L_2-օպտիմալ ռացիոնալ մոտարկման զուգամիտության մասին

Ուսումնասիրվում է L_2-օպտիմալ ռացիոնալ մոտարկման զուգամիտությունը հատվածի ներսում, որտեղ ֆունկցիան ողորկ է: Տեսական գնահատականները ցույց են տալիս, որ զուգամիտության արագությունը՝ համեմատած դասական Կռիլով-Լանցոշի մեթոդի հետ, տարբերվում է կարգով, որը հավասար է ռացիոնալ ֆունկցիայի հայտարարի կարգին:

А. В. Погосян

О сходимости L_2-оптимальной рациональной аппроксимации

Изучается сходимость L_2-оптимальной рациональной аппроксимации внутри отрезка, где функция гладкая. Теоретические оценки показывают, что рациональная аппроксимация точнее классического метода Крылова – Ланцоша и разница в порядке сходимости равна порядку знаменателя рационального аппроксиманта.

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