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Non-Classical State Preparation by Conditional Measurement

(Submitted by the corresponding member G. Yu. Kryuchkyan 26/XI 2007)

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1. **Introduction** As it is known light beams with non-classical states play an important role in the quantum information schemes. This paper discusses the state preparation scheme based on conditional measurement of the correlated subharmonic modes of the non-degenerate optical parametric oscillator (NOPO). The NOPO pumped by a sequence of laser pulses with Gaussian time-dependent envelopes is considered. The intra-cavity non-linear type II parametric interaction realizes the down conversion process which leads to generation of correlated pairs of photons with polarizations orthogonal to each other. According to the method of conditional measurement, since the two subharmonic modes of NOPO are strongly correlated, measurement of the desired state on one of those modes projects the other mode into a similar quantum state. This is performed without actual interaction with the second mode by the measurement. n photons counted in one of the correlated modes projects the other mode in an n-photon Fock state, which can then be analyzed using quantum homodyne tomography. These measurements were recently demonstrated for one-photon Fock state ($n = 1$) [1,2] and for two-photon Fock state ($n = 2$) [3] by using pulsed non-degenerate amplifier producing a pure two-mode squeezed state. The prepared states have been analyzed by the homodyne detection, operating in a time-resolved regime. Here this problem is considered for more general case that includes the full description of dissipative and pump field effects in the framework of the theory of periodically pulsed NOPO [4].

Since the open quantum system is considered for the complete description of the quantum state of the two subharmonic modes of the NOPO, the density matrix $\hat{\rho}$ is used. In the Fock basis it has the following form

$$\hat{\rho} = \sum_{k,l,i,j} \rho_{klij} \left|k;1\right\rangle \left|l;2\right\rangle \left\langle i;1\right| \left\langle j;2\right|, \tag{1}$$

where $\left|n;m\right\rangle$ is the n-photon Fock state of the mode m and ρ_{klij} are the matrix elements.

Separate subharmonic modes can be described by the reduced density matrix, which can be calculated by averaging of the density matrix describing the complete system $\hat{\rho}$ over all states of the other mode

$$\hat{\rho}^{(2)} = \sum_{n} \left\langle n;1\right| \hat{\rho} \left|n;1\right\rangle, \tag{2}$$

or in the matrix form

$$\rho^{(2)}_{nm} = \sum_{k} \rho_{knkm}. \tag{3}$$

It is known that the state of a quantum system can be completely described by the Wigner function. Different properties of quantum states can be easily identified by visual characteristics of the Wigner function. E.g. having negative value range is a convenient indicator of a non-classical state. It can also be measured directly using the quantum homodyne tomography method [5].

The calculations are based on the standard form of the Wigner function in the Fock space

$$W_i(\rho,\theta) = \sum \rho^{(i)}_{nm} W_{nm}(\rho,\theta), (i = 1,2), \tag{4}$$

where ρ, θ are the polar coordinates in the complex phase-space plane and the coefficients $W_{mn}(\rho,\theta)$ are the Fourier transformed functions of matrix elements of the Wigner characteristic function.

2. State preparation schemes. Lets consider that $\left|\xi;1\right\rangle = \sum_n \xi_n \left|n;1\right\rangle$ is the measured state of the first subharmonic mode. The reduced conditional density matrix describing the state of the second mode can be obtained from the density matrix of the complete system in the following way:

$$\hat{\rho}^{(2)}_{\xi} = \frac{\left\langle \xi;1\right| \hat{\rho} \left|\xi;1\right\rangle}{Tr(\left\langle \xi;1\right| \hat{\rho} \left|\xi;1\right\rangle)}. \tag{5}$$

Matrix elements will look like following:

$$\rho^{(2)}_{nm,\xi} = \frac{\sum_{k,l} \xi^*_k \xi_l \rho_{knlm}}{\sum_{k,l,i} \xi^*_k \xi_l \rho_{kili}}. \tag{6}$$

Preparation of two different types of states is considered further. First for the n-photon Fock state ($n = 1,2$) and next for the coherent states.

2.1. Fock states. In case of Fock state $|\xi;1\rangle = |N;1\rangle$, where N is the number of photons of the prepared state, the conditional density matrix will get following matrix form:

$$\rho^{(2)}_{nm}(N) = \frac{\rho_{NnNm}}{\sum_k \rho_{NkNk}}. \tag{7}$$

In Fig. 1 the Wigner functions corresponding to the conditional measurement schemes with $N = 1$ and $N = 2$ photon Fock states are presented. These results are numerically calculated on the base of quantum state diffusion (QSD) approach.

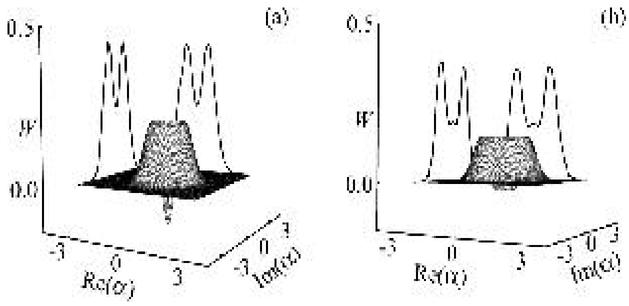

Figure 1: Wigner functions corresponding to the conditional measurement scheme for one-photon (a) and two-photon (b) Fock states.

Both of the Wigner functions clearly display negative regions in the phase-space that reflect a highly non-classical character of quantum states. All Wigner functions are rotationally symmetric and hence the conditional mixed states are phase independent. In Fig. 2 the radial dependence of the Wigner functions is shown. These results are in agreement with experimental and theoretical results of conditional Wigner functions presented in [6].

For the comparison with target states the radial dependence of one-photon and two-photon Fock states are plotted in Fig. 2 with dot lines. The corresponding conditional mixed states are close to Fock states, but demonstrate less non-classicality. Since these non-classical states are a result of quantum interference effect, it is natural that existence of dissipation decreases the level of non-classicality by disrupting the interference.

2.2. Coherent states. In case of coherent state measurement scheme

$$|\xi;1\rangle = |\alpha;1\rangle, \qquad \xi_n = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \tag{8}$$

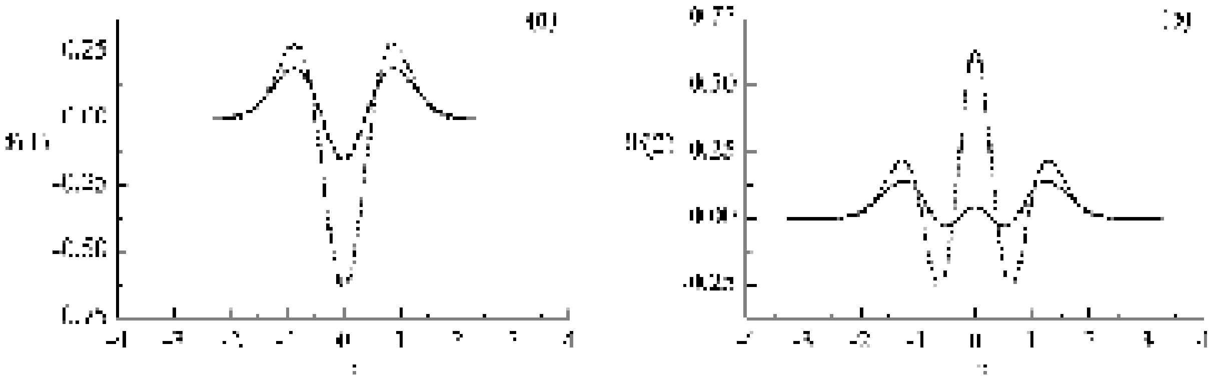

Figure 2: Radial dependence of Wigner functions corresponding to the conditional measurement scheme for one-photon (a) and two-photon (b) Fock states. Wigner functions corresponding to Fock states are plotted in dot lines for comparison.

where α is complex number describing the coherent state, the conditional reduced density matrix will have the following matrix from:

$$\rho^{(2)}_{nm}(\alpha) = \frac{\sum_{l,k} \frac{\alpha^{*l}\alpha^{k}}{\sqrt{l!k!}} \rho_{lnkm}}{\sum_{l,k,i} \frac{\alpha^{*l}\alpha^{k}}{\sqrt{l!k!}} \rho_{liki}}. \tag{9}$$

Resulting Wigner functions are plotted in Fig. 3. Due to above mentioned reasons these states are close but do not exactly match coherent states. These calculations also show that the center of the resulting states is equal to complex conjugate values of the center points α of the measured coherent state.

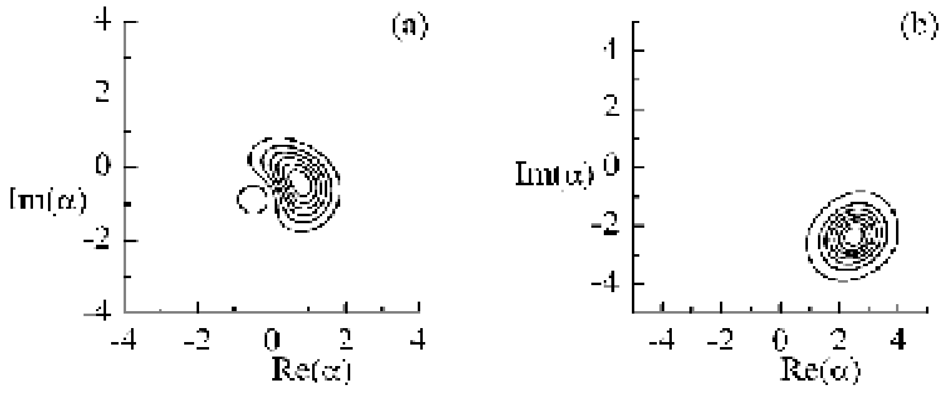

Figure 3: Contour plots of Wigner functions corresponding to the conditional measurement scheme with coherent states for intensity of pump field below (a) and above (b) of the generation threshold.

The calculations are done for both below threshold (Fig. 3a) and above threshold (Fig. 3a) regimes. It should be noted that in the case of below threshold regime, there is a region in the phase space where the Wigner function is negative, which disappears when the intensity if the pump filed is increased.

In the case of below threshold regime, there is a region in the phase space where the Wigner function is negative, which indicates the existence of quantum interference. In the example Wigner function plotted in Fig. 3a this region is located around $(-0.5, -1)$ point. This effect disappears when the intensity (if the pump filed) is increased, and the corresponding Wigner functions get form close to Gaussian, which corresponds to coherent states.

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Non-Classical State Preparation by Conditional Measurement

The present paper is devoted to the investigation of quantum state preparation problem. This is a method of producing non-classical quantum states, which is based on down conversion process in non-linear type II crystals. The non-degenerate optical parametric oscillator (NOPO) pumped by a sequence of laser pulses with Gaussian envelopes is considered. The idea standing behind this scheme is that the subharmonic modes of NOPO are strongly correlated and the measurement of the desired state on one of those modes projects the other mode into a similar state, meantime without actually interacting with it by measurement. Two different target state types are considered: Fock states (exact photon number states) and coherent states. The analysis is done in the framework of Wigner quasi-probability distributions in the phase space.

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Исследована проблема приготовления квантовых состояний на примере процесса вниз конверсии, который осуществляется обычно в нелинейных кристаллах II типа. Рассмотрен невырожденный оптический параметрический генератор (НОПГ) под действием последовательности лазерных импульсов с гауссовской временной

огибающей. Идея этой схемы состоит в том, что субгармонические моды НОПГ сильно коррелированы и измерение целевого состояния на одной из мод проектирует другую моду в похожее состояние. В работе рассмотрены два разных вида целевых состояний - фоковские состояния (состояния с конкретным числом фотонов) и когерентные состояния. Анализ произведен в рамках квази-вероятностных распределений Вигнера в фазовом пространстве.

Ն.Հ. Ադամյան

Ոչ դասական վիճակների պատրաստումը պայմանական չափումների միջոցով

Աշխատանքը նվիրված է քվանտային վիճակների պատրաստման խնդիրների ուսումնասիրությանը: Սա ոչ դասական քվանտային վիճակների ստեղծման մեթոդ է՝ հիմնված II-րդ սեռի ոչ գծային բյուրեղներում փոտոնների տրոհման պրոցեսի վրա: Դիտարկված է ժամանակային զաուսյան պարփակող կորագծով ամփոփված լազերային իմպուլսների հաջորդականության ազդեցության տակ գտնվող ոչ այլասերված օպտիկական պարամետրիկ գեներատոր (ՈՕՊԳ): Այս սխեմայի հիմքում ընկած է այն, որ ՈՕՊԳ-ի սուբհարմոնիկ մոդերը ուժեղ կոռելյացված են, որի հետևանքով մոդերից մեկի վրա կատարված նպատակային վիճակի չափումը երկրորդ մոդը արտապատկերում է համանման վիճակի՝ առանց չափման միջոցով փոխազդման: Աշխատանքում դիտարկված են երկու տիպի նպատակային վիճակներ՝ Ֆոկի (որոշակի ֆոտոնային թվով) և կոհերենտ: Հետազոտությունը կատարված է փուլային տարածությունում Վիգների քվազի-հավանականային բաշխումների շրջանակներում:

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