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Academician S. A. Ambartsumian, M. V. Belubekyan, K. B. Ghazaryan

Some Aspects of Space Elevator Ribbon Stress and Length Reduction

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1. **Statement of the problem.** Let us consider a very long elastic ribbon anchored on the Earth equatorial point [1-3]. The ribbon subject to the action of Earth gravity inward force F_1, defined by Newton gravity law and centrifugal outward force, due to Earth daily spinning.

$$F_1(\gamma) = \frac{\rho g_0 R_0^2}{(R_0+\gamma)^2}; F_2(\gamma) = -\rho\omega^2(R_0+\gamma). \qquad (1)$$

In Eq. (1) γ is a coordinate along ribbon length counted from Earth surface, ρ is the bulk density of the ribbon material, $R_0 = 6378$km is the Earth equatorial radius, $g_0 = 980$sm·sec^{-2} is the gravity force acceleration on the Earth surface, $\omega = 2\pi/T$ is the circular spinning frequency of the Earth, $T = 86146$sec is the period of the Earth spinning.

When the cross-section area $S(x)$ of the ribbon is the function of its length, using the dimensionless notations the equation determining the elastic stress $\sigma(x)$ can be written as [2]

$$\frac{d[s(x)\sigma(x)]}{dx} + \rho g_0 R_0 g(x) S(x) = 0, \qquad (2)$$

$$g(x) = \alpha(1+x) - (1+x)^{-2}.$$

Here the following dimensionless quantities are used:

$$x = \gamma/R_0,\ L = l/R_0,\ \alpha = \omega^2 R_0/g_0 \approx 1/288.$$

Function $g(x)$ has the following properties

$$g(x) > 0 \text{ at } x < x_0,\ g(x) < 0 \text{ at } x > x_0,\ x_0 \approx 5,6,$$

where the point x_0 corresponds to the Earth geosynchronous orbit.
Eq. (2) is to be considered with the following boundary condition at free end $x = L$

$$\sigma(L) = 0 \tag{3}$$

and condition

$$\sigma(0) = 0. \tag{4}$$

The condition (4) ensures tension stresses $\sigma(x) \geq 0$ at any point along of ribbon and defines the limiting length L_0 of tensile ribbon.
When $S(x) = \text{const}$, the elastic stress is defined by the following function, satisfying Eqs. (2-4)

$$\sigma(x) = \rho g_0 R_0 \tilde{\sigma}_0(x),$$

$$\tilde{\sigma}_0(x) = \frac{(L_0 - x)[(1 + L_0)(1 + x)(2 + L_0 + x) - 576]}{576(1 + L_0)(1 + x)}. \tag{5}$$

Here $L_0 \approx 22.45$ is the limiting length of tensile ribbon. If ribbon length is less than limiting length L_0, then the compression stresses arise localized near ribbon base and due to it, the ribbon may become unstable Ref.(3).
The mechanical tension stress reaches its maximum value $\sigma_0 \approx 0.78\sigma g_0 R_0$, at point $x_0 \approx 5.602$

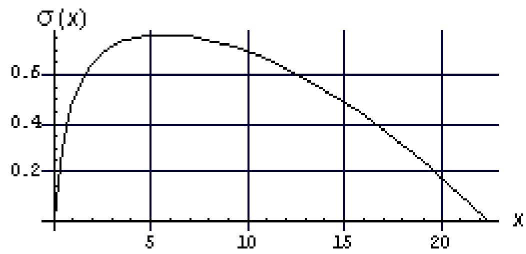

Figure 1. The stress function for the uniform ribbon

2. Non-uniform Ribbon. Based on the results presented in the above section the following problem will be considered: by bulging the ribbon cross section in the neighbourhood of point x_0 to minimize the maximum value of stress function of the ribbon of length L_0, defined by Eq. (4), under restriction $\sigma(x) \geq 0$.
For this purpose the model of ribbon with the following step-piece homogeneous cross-section will be studied

$$S(x) = \begin{cases} S_1, & 0 \geq x < l_1 < x_0, \\ S_2, & l_1 \geq x \geq L_0 - l_2, \\ S_1, & x_0 < L_0 - l_2 < x \geq L_0. \end{cases} \tag{6}$$

Integrating the Eq.(2) we come to the following stress functions ($F(x) = \rho g_0 R_0 g(x)$)

$$\sigma(x) = \begin{cases} \sigma_1(x) \\ \sigma_2(x) \\ \sigma_3(x) \end{cases} = \begin{cases} -\int\limits_0^x F(\xi) d\xi; & 0 \geq x < l_1, \\ -\int\limits_{l_1}^x F(\xi) d\xi + C_1; & l_1 \geq x \geq L_0 - l_2, \\ -\int\limits_{L_0 - l_2}^x F(\xi) d\xi + C_2; & L_0 - l_2 < x < L_0. \end{cases} \tag{7}$$

Arbitrary constants C_1 and C_2 are determined from the conditions of the equality of stress resultants at $x = l_1$, $x = L + 0 - l_2$

$$S_1 \sigma_1(l_1) = S_2 \sigma_2(l_1); \; S_2 \sigma_2(L_0 - l_2) = S_1 \sigma_3(L_0 - l_2) \tag{8}$$

or

$$\begin{aligned} -S_1 \int\limits_0^{l_1} F(x) dx = S_2 C_1; \\ S_2 \left[- \int\limits_{l_1}^{L_0 - l_2} F(x) dx + C_1 \right] = S_1 C_2. \end{aligned} \tag{9}$$

In the interval $l_1 \geq x \geq L_0 - l_2$ the maximum value of stress is approached at the point $x = x_0$, where $F(x_0) = 0$

$$\max_x \sigma_2(x) = \sigma_2(x_0). \tag{10}$$

In the intervals $0 \leq x \leq l_1$, $L_0 - l_2 \leq x \leq L_0$ the maximum value of the stresses is reached at end points l_2, $L_0 - l_2$

$$\max_x \sigma_1(x) = \sigma_1(l_1); \; \max_x \sigma_3(x) = \sigma_3(L_0 - l_2). \tag{11}$$

The optimization problem is to choose parameters l_1, l_2 in such a way, so that to minimize the greatest from maximum values of Eqs. (10,11).
Let us show that the optimization condition will be fulfilled if

$$\sigma_1(l_1) = \sigma_2(x_0) = \sigma_3(L_0 - l_2). \tag{12}$$

From equality $\sigma_1(l_1) = \sigma_3(L_0 - l_2)$ it follows that

$$\int\limits_{l_1}^{L_0 - l_2} g(x) dx = 0 \tag{13}$$

or

$$L_0 - l_2 = \frac{1}{2}\left[\sqrt{(1+l_1)^2 + \frac{8}{\alpha(1+l_1)}} - 3 - l_1\right]. \tag{14}$$

As the condition $\sigma_1(l_1) = \sigma_2(x_0)$ does not depend from parameter l_2, so the equality (14) can be considered as a formula defining l_2 by means of given values of l_1. From Eqs .(8,9) the expressions $\sigma_1(l_1)$, $\sigma_2(x_0)$ can be written as ($\gamma = S_1/S_2 < 1$)

$$\begin{aligned}
&\sigma_1(l_1) = \rho g_0 R_0 \frac{l_1}{1+l_1}\left[1 - \alpha\left(1 + \frac{l_1}{2}\right)(1+l_1)\right], \\
&\sigma_2(x_0) = \rho g_0 R_0 \left\{\frac{x_0 - l_1}{(1+x_0)(1+l_1)}\left[1 - \alpha\left(1 + \frac{x_0+l_1}{2}\right)(1+x_0)(1+l_1)\right] + \right. \\
&\qquad \left. + \gamma\frac{l_1}{1+l_1}\left[1 - \alpha\left(1+\frac{l_1}{2}\right)(1+l_1)\right]\right\}.
\end{aligned} \tag{15}$$

From the Eqs. (12) the following equation can be obtained

$$\begin{aligned}
f(l,\gamma) \equiv \frac{x_0 - l_1}{1+x_0}\left[1 - \alpha\left(1 + \frac{x_0+l_1}{2}\right)(1+x_0)(1+l_1)\right] - \\
-(1-\gamma)l_1\left[1 - \alpha\left(1+\frac{l_1}{2}\right)(1+l_1)\right] = 0.
\end{aligned} \tag{16}$$

Derivative $\sigma_1(l_1)$ respect to l_1 is determined as

$$\frac{\partial\sigma_1(l_1)}{\partial l_1} = \frac{1 - \alpha(1+l_1)^3}{(1+l_1)^2} > 0 \quad (l_1 < x_0) \tag{17}$$

i.e. $\sigma_1(l_1)$ is an increasing function.
The derivative of function $\sigma_2(x_0)$ respect to l_1 is a negative one, i.e. $\sigma_2(x_0)$ is a decreasing function.
So, the optimization problem comes to the solution of Eq.(17) defining the parameter l_1 and as a result the minimum value of $\max\sigma(x)$ with respect to parameter γ. Eq. (16) has only one real root in the interval $0 < l_1 < x_0$.

Table 1

Numerical results for optimal ribbon

γ	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
$\tilde{\sigma}_1$	0.782	0.704	0.646	0.595	0.553	0.516	0.484	0.456	0.430	0.408	0.39
$\tilde{\sigma}_2$	0.782	0.742	0.701	0.663	0.624	0.585	0.546	0.507	0.468	0.429	0.39
l_1	5.6	2.643	1.930	1.533	1.276	1.093	0.957	0.851	0.767	0.698	0.640
l_2	16.9	12.616	10.867	9.634	8.818	7.924	7.295	6.761	6.273	5.911	3.741

In Table 1, based on the solution of Eq. (16) the numerical results for l_1, l_2 and appropriate corresponding values of maximal stress $\tilde{\sigma}_1 = \max \sigma(x)/\rho g_0 R_0$ are given for different values of parameter γ (the second row of Table 1.)

As it follows from the data of Table 1, the bulging of the ribbon at the point of the Earth's geosynchronous orbit leads to the decreasing of the maximum tension stress. Particularly, if cross-section area in the interval containing point x_0 increasing ten times, the maximum stress will be reduced by two times.

Let us consider now the other case of the ribbon bulging at the Earth's geosynchronous orbit.

Let us take the ribbon cross section as function similar to the uniform ribbon stress function

$$S(x) = S_0 + S_1 \sigma_0(x) > 0. \tag{18}$$

In this case based on solution of Eq. (2) the stress function can be written as $(\alpha = S_1/S_0)$

$$\sigma(x) = \frac{\rho g_0 R_0 \int\limits_x^L g(\eta)[1+\alpha\sigma_0(\eta)]d\eta}{1+\alpha\sigma_0(x)}. \tag{19}$$

Taking into account that

$$\frac{d\sigma_0(\eta)}{d\eta} = -g(\eta),$$

we come to

$$\sigma(x) = \frac{\rho g_0 R_0 \left[\sigma_0(x) - \alpha \int\limits_x^L \sigma_0(\eta)\frac{d\sigma_0(\eta)}{d\eta}d\eta\right]}{1+\alpha\sigma_0(x)} = \rho g_0 R_0 \sigma_0(x)\left[\frac{2+\alpha\sigma_0(x)\alpha}{2(1+\alpha\sigma_0(x))}\right]. \tag{20}$$

This function takes its maximum value at point x_0

$$\tilde{\sigma} = \frac{\max\sigma(x)}{\rho g_0 R_0} = \sigma_0(x_0)\left[\frac{2+\alpha\sigma_0(x)\alpha}{2(1+\alpha\sigma_0(x))}\right]; \quad (\sigma_0(x_0) = 0.78). \tag{21}$$

Based on formula Eq.(21) the values of the maximum stress value $\tilde{\sigma}_2$ depend upon $\gamma = S(0)/S(x_0)$ is given in the third row of Table 1.

The above presented results determine the optimal design projects of the ribbon when the "limit" length does not change, while the ribbon forms permit the decrease of the maximum stress.

3. Optimal Problem for a Ribbon Made From Non-Homogeneous Material. Let us consider the ribbon made from a non-homogeneous material, namely when the density of the ribbon material is a function of its length. For this ribbon we would

like to formulate the following optimal problem, the solution of which is important from the applied point of view. Considering the density $\rho(x)$ of the ribbon as a variable of the design and the mechanical stress as a function of the objective, such a ribbon should be defined, for which $\max\limits_{x} \sigma(x) \to \min\limits_{\rho(x)}$, $\tilde{\rho}_{01} < \rho(x) < \tilde{\rho}_{02}$, under restriction $\sigma(0) = 0$.
Based on Eq.(3) we have the following solution for the stress function

$$\sigma(x) = g_0 R_0 \int\limits_{x}^{L} \rho(\eta) g(\eta) d\eta, \tag{22}$$

Obviously, that

$$\max_{x} \sigma(x) = g_0 R_0 \int\limits_{x_0}^{L} \rho(\eta) g(\eta) d\eta. \tag{23}$$

Taking into account restriction we have

$$\max_{x} \sigma(x) = g_0 R_0 \int\limits_{0}^{x_0} \rho(\eta) |g(\eta)| d\eta. \tag{24}$$

From Eq.(24) it follows that function

$$\rho(x) = \begin{cases} \rho_1; & x \in [0, x_0]; \\ \rho_2; & x \in [x_0, \tilde{L}_0] \end{cases} \tag{25}$$

is the optimal solution of the problem under consideration.
The minimum value of $\max\limits_{x} \sigma(x)$ will be defined as

$$\tilde{\sigma}_0 = 0.78 \rho_1 g_0 R_0, \tag{26}$$

while the limiting length of tensile ribbon $\tilde{L}_0$ will be defined from the following equation

$$\rho_2 \int\limits_{x_0}^{\tilde{L}_0} g(\eta) d\eta = 0.78 \rho_1. \tag{27}$$

Based on Eq.(27) numerical results for the limiting length of the tensile ribbon $\tilde{L}_0$ with respect to the parameter $\beta = \rho_1 / \rho_2$ is presented in the Table 2.

Table 2

Numerical results for optimal ribbon length

β	1	0.9	0.7	0.5	0.3	0.1	0.05	0.01
L_0	22.4	21.7	19.54	17.1	14.2	10.2	8.74	6.9

4. Conclusions. Based on the solution of the one- dimension equation of the elasticity theory qualitative and quantitative results are obtained related to the strength of a space elevator cable/ribbon. It is assumed that the ribbon subjected to the action of the Earth gravity inward force, defined by Newton gravity law and the centrifugal outward force, due to the Earth's daily spinning. We confined ourselves to results related to the strength problems of the ribbon bulged in neighbourhood of point of the Earth's geosynchronous orbit. The optimal design problem of the ribbon made from non-homogeneous material when the density of ribbon material is the function of its length is also considered. It is shown that the optimal project is the compound ribbon made from two homogeneous materials.

On the other hand, there is no doubt that the future objects of the investigation should be constructions of elastic closed shell (pipe) type, which are of interest from an applied point of view. For such constructions, the new mechanical problems should be considered, which take into account circular, transversal stresses and displacements arising in the shells. Among these problems we can list the dynamic interaction of closed shells subject to external media, including electromagnetic, temperature and atmosphere fields' actions.

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National Academy of Sciences of Armenia

Academician S. A. Ambartsumian, M. V. Belubekyan, K. B. Ghazaryan

Some Aspects of Space Elevator Ribbon Stress and Length Reduction

The possibility of the realization and exploitation of the space elevator project is connected with a number of complicated problems. Some of them are large stresses arising in the space elevator ribbon body, which are considerably bigger that the limit of strength of modern materials. This paper is devoted to the solution of problem of maximum stress reduction in the ribbon by the modification of the ribbon cross-section area. The aspects of the ribbon length reduction are also considered.

Ակադեմիկոս Ս. Ա. Համբարձումյան, Մ. Վ. Բելուբեկյան, Կ. Բ. Ղազարյան

Տիեզերական վերելակի ճոպանի երկարության և լարումների նվազեցման հարցեր

Տիեզերական վերելակի նախագծի իրականացման և շահագործման հնարավորությունը բազմաթիվ բարդ գիտատեխնիկական խնդիրների լուծման հետ է կապված: Այդպիսի խնդիրներից մեկը տիեզերական վերելակում մեծ լարումների առաջացումն է, որոնք մի կարգ ավելի մեծ են ժամանակակից նյութերի ամրության սահմանից:

Հոդվածը նվիրված է վերելակի ճոպանի ընդլայնական հատույթի մակերեսի փոփոխության միջոցով մաքսիմալ լարման նվազման խնդրի լուծմանը: Քննարկված են նաև ճոպանի երկարության նվազեցման հարցերը:

Àêàäåìèê Ñ. À. Àìáàðöóìÿí, Ì. Â. Áåëóáåêÿí, Ê. Á. Êàçàðÿí

Âîïðîñû óìåíüøåíèÿ íàïðÿæåíèé è äëèíû òðîñà êîñìè÷åñêîãî ëèôòà

Возможность реализации и эксплуатации проекта космического лифта связана с решением большого числа сложных научно-технических проблем. Одной из этих проблем является возникновение больших напряжений в теле космического лифта, которые на порядок превышают предел прочности современных материалов.

Статья посвящена решению задачи уменьшения максимального напряжения путем модификации площади поперечного сечения троса лифта. Обсуждены также вопросы уменьшения длины троса.

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